Bell's Theorem
Hidden Variables and Quantum Reality

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Abstract

We explore the Einstein-Podolsky-Rosen paradox, the idea of hidden variables, and John Bell's proof that the existence of those variables in any local physical theory, regardless of what those variables are or how they behave, leads to a prediction measurably different from the prediction offered by quantum mechanics. Finally, an examination of the experiments of Aspect et al. and Weihs et al. confirms the reality of quantum mechanics.

1 Introduction

When first introduced to quantum mechanics, a typical student may understand the mathematics and grasp the framework within which one performs calculations. Very rarely, though, does a student immediately internalize the oddities of quantum theory. Many struggle with the question of the interpretation. The probabilistic interpretation is often initially understood as simply a lack of information - this mistake can arise from preconceived notions due to an encounter with Schrödinger's cat or other popular examples where superposition seems like an impossibility.

This paper will serve as an accessible reinforcement of the truth: quantum mechanics is not missing anything. It is not that we cannot *know* simultaneously position and momentum, it is that in actuality position and momentum do not simultaneously coexist. It is not that we cannot *know* simultaneously the spin of an electron in multiple directions, it is that in actuality an electron's spin is only defined along one axis at a time. Moreover, it matters experimentally whether the probabilities represent just our lack of information or a fundamental aspect of nature.

We will discuss spacelike separation and its implications for communication, the paradox of Einstein, Podolsky, and Rosen and the implications of local hidden variable theories, and the discovery of John Bell: it matters experimentally if the assumptions leading to that paradox are true. We will discuss experiments which have resulted in measurements like those suggested by Bell, their affirmation of quantum mechanics, and their exclusion of local hidden variable theories.

2 Special Relativity And Spacelike Separation

Einstein's postulation that the speed of light, c, is agreed upon by all inertial observers leads directly to the well-known Lorentz transformations. Furthermore, we discover three fundamentally different kinds of separation in what is known as space-time. One finds that the space-time displacement Δs between the origin O and the point $P = (ct, x, yz)$ is

$$\Delta s^2 = -c^2t^2 + x^2 + y^2 + z^2 \ . \tag{1}$$

From this equation, we see three different possibilities for Δs^2.

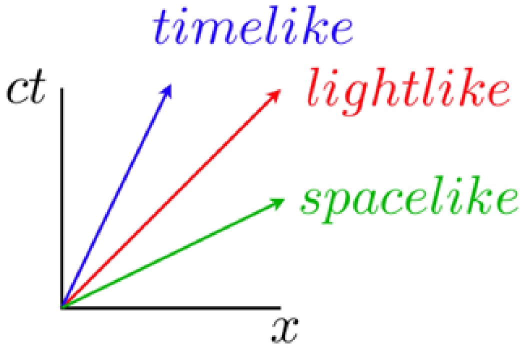

Figure 1: The three different kinds of separation in a two dimensional spacetime.

If Δs^2 is negative, we say that P is *timelike* separated from the origin. This means that regardless of your velocity, P will always happen later in time than O. If Δs^2 is zero, we say that P is *lightlike* separated from O. This means that regardless of your velocity, you could see the same photon travel through the origin and then through P.

Finally, if Δs^2 is positive, we say that P is *spacelike* separated from the origin. This means that regardless of your velocity, you will never see anything traveling at light speed or slower move from O to P. Nothing, not even light, can make the journey quickly enough. Suppose that in a particular inertial frame, P occurred later O, yet the two were still spacelike separated. Since they are spacelike, one can always find an inertial frame where O and P are simultaneous, or even an inertial frame in which O occurs later than P. Thus, if in the original frame what happened at O influenced what happened at P, in the new frame we will see the effect occur before the cause.

Spacelike separation is of particular interest to us. If we could send influence across a spacelike separation something strange would be happening indeed. Communication across such a separation, is prohibited: it is not possible to send a message into the past. Einstein used this idea of a speed limit on causality to attack quantum mechanics.

3 The EPR Paradox

In 1935, Einstein, Podolsky, and Rosen (EPR) published a paper arguing that, while quantum mechanics is a useful theory which successfully predicts the results of experiments, it cannot be the end of the story: it must be missing something. They asserted that "Every element of the physical reality must have a counterpart in the physical theory." Furthermore they suggest that

> If, without in any way disturbing a system, we can predict with its certainty (i.e. with a probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity.[1]

This seems entirely reasonable—if you do not do anything to a particle yet deduce something about it, it must have been that way before your efforts.

Let us consider the implications of this *reality condition* for the example put forward by Bohm and Aharonov [2]. Bohm and Aharonov considered a system of two spin-$\frac{1}{2}$ particles, A and B, in the spin singlet state. If we denote the state of the system as $|\Psi\rangle$, a spin up state with the label $+$, and a spin down state with the label $-$, the spin singlet state may be written as

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\left(|+\,-\rangle - |-\,+\rangle\right) \tag{2}$$

where the first and second labels correspond to the states of A and B, respectively. Recall that the spin singlet state has zero total angular momentum. This liberates us from choosing the axis along which we measure the angular momentum, because two particles which have no net angular momentum should have opposite spins whether measured along the z-axis, x-axis, or any other axis of our choosing. Thus, Eq. 2 describes the spin singlet state in a completely general way, regardless of the axial basis we choose.

In quantum mechanics, a spin-$\frac{1}{2}$ particle has a well-defined spin in only one direction at a time. If we know a particle to be spin-up along z we can say it is described by the ket $|+z\rangle$. However, we cannot say that it simultaneously has a definite spin along x. To describe the results of one particular Stern-Gerlach experiment, we can write

$$|+z\rangle = \frac{1}{\sqrt{2}}\left(|+x\rangle + |-x\rangle\right) \tag{3}$$

$$|-z\rangle = \frac{1}{\sqrt{2}}\left(|+x\rangle - |-x\rangle\right)\ . \tag{4}$$

Measuring the spin along x of $|+z\rangle$, forces the particle to choose $|+x\rangle$ or $|-x\rangle$. Then, by solving for $|+x\rangle$ (or $|-x\rangle$) we find the particle is no longer in $|+z\rangle$: it is in a superposition of $|+z\rangle$ and $|-z\rangle$. Quantum mechanics thus prevents us from describing a particle with definitive spin in more than one direction. It is nonsense to write

$$|+z\ ;\ -x\rangle \tag{5}$$

because if the particle has a down spin along x, we cannot specify its spin along z, one way or the other. Quantum mechanics accomplishes this by describing the measurement along z, S_z, and the measurement along x, S_x, as two operators which do not commute:

$$[S_z\ ,\ S_x] = i\hbar S_y \neq 0\ . \tag{6}$$

We find, more generally, that spin measurements along two different axes do not commute.

Now, let us suppose that we separate the two particles in the spin singlet state without measuring their angular momenta. Suppose we send B far away and keep particle A with us, at the origin, winding up with both particles at rest in the laboratory frame. Now, at any given instant in the lab frame, the two particles are spacelike separated, incapable of interacting—if measurements of these particles take place simultaneously in the lab frame, they must be spacelike separated. If we then measure A to be, for instance, up along the z-axis, we know immediately that B is spin-down along that axis—the system that was described by Eq. 2 collapsed into $|+\,-\rangle$. Since the news of the measurement of A's spin could in no way reach B and disturb it instantaneously, yet we have determined B's z-spin, the EPR assertions suggest that B's spin along the z-axis exists as an element of physical reality independent of our measurement. This would imply that the wavefunction $|\Psi\rangle$ did not tell the whole story—we only described the system in a superposition because we didn't *know* if it was $|+\,-\rangle$ or $|-\,+\rangle$, but in reality we know that B was in the down state all along, and thus the system was *in reality* definitively $|+\,-\rangle$. EPR would argue that quantum mechanics is not complete, because it could not tell us that the system was definitively in $|+\,-\rangle$.

The implications for quantum mechanics worsen. Instead of measuring along the z-axis, we could have, in principle, measured A's spin along the x-axis, or any other axis, to determine B's spin along that axis without disturbing it. Thus, B's spin in each direction must be an element of physical reality, meaning that B truly does have a physically realized spin along multiple axes at a time. If only considering the z- and x-axes, a description of the physical state like Eq. 5 becomes necessary, though quantum mechanics declares it meaningless. Moreover, if we consider the infinite number of axes along which we could measure the spin, in order to describe the state of a single spin-$\frac{1}{2}$ particle fully, we would need a ket with an infinite number of labels.

Notice that the EPR paradox does not conflict with the uncertainty principle. Einstein and his colleagues fully recognized that if you did indeed measure the spin of A along the x-axis the spin of A along the z-axis would be disturbed. However, the EPR argument goes, B still has a definite z-axis spin because we could have instead measured along the z-axis (in which case the x-axis spin would be disturbed). Since we could do this experiment by measuring either particle, A must have physically realized spins along the z- and x-axes as well, but we cannot measure one without disturbing the other. As the uncertainty principle is a major result in quantum mechanics, this inability to conquer it and describe the spins in all directions shows a fundamental limit of the theory.[9]

If the reality conditions of Einstein, Podolsky, and Rosen are true, it is a disaster for the theory of quantum mechanics. It reduces quantum theory from an accurate theory which describes an inherently probabilistic nature to a theory which is probabilistic due to the lack of information. If the assertions are true, then there is a deeper theory awaiting discovery. The EPR paper is concluded with the following statement:

> While we have thus shown that the wave function does not provide a complete description of the physical reality, we left open the question of whether or not such a description exists. We believe, however, that such a theory is possible.[1]

This suggests that there are additional quantities which are required to explain physical law at the truly fundamental level. Maybe the quantum formalism is not the final formalism. Perhaps the theory that tells full story reproduces the effects of the uncertainty principle but gets around being probabilistic in nature through other ideas not yet conceived.

4 Hidden Variables

EPR did not dispute the results of repeatable experiments—they fully admitted that a Stern-Gerlach setup which measured S_z, discarded the $|{-z}\rangle$ particles, measured S_x, discarded the $|{-x}\rangle$ particles, and finally remeasured $S_z$would find the unintuitive result of the reappearance of $|{-z}\rangle$ particles. However, the sentiment was, perhaps there were *hidden variables* which we do not know about which relieve our physical theory from being merely probabilistic. These hidden variables might be measurable, or they might be unobservable in principle, or there might be some of the first type and some of the second. Either way, nobody at the time could say what those variables would be, how to calculate them, how they evolved in time (if they did at all), or how they interacted with the currently known observables.

Wolfgang Pauli pointed out that such a description of reality was outside of the realm of physics. He said, "One should no more rack one's brain about the problem of whether something one cannot know anything about exists all the same, than about the ancient question of how many angels are able to sit on the point of a needle."[9] This comment conveys the idea that physics is only concerned with observable quantities. Additionally, the remark is pointed towards Einstein, who many years earlier dismissed the luminiferous ether as superfluous by removing from it the effects now understood to arise from the invariance of the speed of light—the ether was not tangible, and Einstein's insight led people to give it up as a part of the description of the universe.[9] When faced with an argument comparing the uncertainty natural in quantum mechanics to the consequenceless ether, Einstein once said, "A good joke should not be repeated too often."[10] Einstein felt that even if physicists could not break through the uncertainty barrier with their observations, it was still their task to determine exactly what laws governed the universe.

In 1964, John Bell discovered that the very existence of those hidden variables, regardless of what they were or how the behaved, led to a definite prediction which differed from the quantum mechanical prediction.

5 An Example

Let us work through an instructive example. Suppose two observers Alice and Bob are spacelike separated and agree to measure respectively the spins of particles A and B which are in the spin singlet state. For simplicity, Alice and Bob constrain themselves to only measuring along the three coordinate axes. When Alice observes A to be spin up she records a $+1$ and when she observes A to be spin down she records a -1, and Bob does the same. Alice and Bob agree to do a *lot* of repetitions of this experiment, so that their observations correspond with the probabilities given by the wavefunction.

Alice and Bob know that their particles are in the spin singlet state, and that if they measure along the same axis, they will always record one $+1$ and one -1 between them, for a total angular momentum of zero. However, if they measure along different axes, what do they expect?

Let us first assume quantum mechanics gives the accurate description of this system. If Alice measures along z and records $+1$, when Bob measures along x, half the time he will record $+1$ and half the time he will record -1, as suggested by Eq. 3. Similarly, if Alice measures along z and records -1, Bob will also record each possibility an equal amount. We say Alice and Bob "agree" if the numbers they record multiply to -1. In a quantum theory, Alice and Bob will agree half the time if they measure along different coordinate axes.

Suppose instead the system was described as EPR seems to demand. Restricting ourselves to the coordinate axes, we could write a particle's "program" which determines its spin in each direction, in a similar fashion to Eq. 5. For example, a particle whose spins along the x-, y-, and z-axes are up, up, and down respectively can be denoted by

$$|-z\ ;\ +y\ ;\ +x\rangle\ . \tag{7}$$

Since A and B are in the spin singlet state, if A is in $|-z\ ;\ +y\ ;\ +x\rangle$, B must be in $|+z\ ;\ -y\ ;\ -x\rangle$. If Alice measures along x she will absolutely find that A is spin up. If Bob measures B's spin along y, he will absolutely find spin down. For this pair of states, all of the possible outcomes are shown in Table 1.

Table 1: A summary of the results of all possible combinations of measurements by Alice and Bob if the state of A is $|-z\ ;\ +y\ ;\ +x\rangle$. Each entry is the product of what Alice records and what Bob records. The diagonal elements are left out, as those measurements do not distinguish a quantum mechanical theory from a hidden variable theory.

		Alice		
		x	y	z
Bob	x		-1	$+1$
	y	-1		$+1$
	z	$+1$	$+1$	

Notice that two of the six entries are -1. It is easy to convince yourself that this feature only depends on the fact that two of the labels on A were the same. If A had been in the state $|+z\ ;\ +y\ ;\ +x\rangle$ and B in $|-z\ ;\ -y\ ;\ -x\rangle$ or vice-versa, it is obvious that all entries would be -1. Thus, at worst (if there are no particles in $|+z\ ;\ +y\ ;\ +x\rangle$ or $|-z\ ;\ -y\ ;\ -x\rangle$) Alice and Bob agree one third of the time when they measure different axes, which is experimentally distinguishable from agreeing half the time, as they would in a quantum mechanics.

This is deep. Simply assuming the EPR assertions are true leads to a prediction that can be used to determine which kind of theory correctly describes our universe. This is surprising: EPR's assertions were largely philosophical, yet they directly lead to a quantifiable, testable prediction. We will discuss this surprising fact further.

6 Bell's Theorem

One can show that in quantum mechanics $P(\vec{a}, \vec{b})$, the average value of the product of what Alice records when she measures along $\vec{a}$ and what Bob records when he measures along $\vec{b}$, is given by

$$P(\vec{a}, \vec{b}) = -\vec{a} \cdot \vec{b} \tag{8}$$

which becomes the familiar result if either $\vec{b} = \vec{a}$ or if $\vec{b} \perp \vec{a}$. Bell proved that a hidden variable theory that obeys the locality restrictions of EPR is inconsistent with that formula. The proof requires a little analysis.

Suppose our hidden variable theory yields two functions $\alpha(\vec{a}, \lambda)$ and $\beta(\vec{b}, \lambda)$ which take $\vec{a}$ and $\vec{b}$ as arguments, respectively, and additionally depend on the hidden variables λ, which tells us what Alice will record when she measures particle A and what Bob will record when he measures particle B, respectively. We call our theory local because α is not a function of $\vec{b}$, nor β a function of $\vec{a}$, whose choosing could be spacelike separated from the measurements those functions describe.

We know experimentally that these functions can only take on the values of ± 1. We treat λ as though it were a single-valued continuous parameter because, as Bell pointed out, it makes no difference whether it is a single variable, a set of variables, a set of functions, or a combination of these because we intend to always integrate over λ—writing a double integral or triple integral or however many integrals you please will stay consistent throughout the proof, and λ never comes into play otherwise. Suppose λ was distributed with a normalized, nonnegative probability density function $\rho(\lambda)$. The information about $\rho(\lambda)$ is what we are missing and what we can also assume a hidden variable theory provides.

We know that when Alice records $+1$ Bob must record -1 and vice versa if they measure along the same axis. This means that regardless of λ

$$\alpha(\vec{a}, \lambda) = -\beta(\vec{a}, \lambda) \ , \tag{9}$$

and we eliminate β from our discussion. Now we can calculate $P(\vec{a}, \vec{b})$. Writing the expression is easy.

$$\begin{aligned} P(\vec{a}, \vec{b}) &= \int d\lambda \rho(\lambda) \alpha(\vec{a}, \lambda) \beta(\vec{b}, \lambda) && (10) \\ &= -\int d\lambda \rho(\lambda) \alpha(\vec{a}, \lambda) \alpha(\vec{b}, \lambda) \ , && (11) \end{aligned}$$

where the second line follows the first by substituting in Eq. 9. We would like to analyze how P responds to a change of $\vec{b}$, and formally this can done via a rigorous

ϵ-δ style proof. We will proceed in a more accessible manner. To see how P responds to a change of $\vec{b}$, it is natural to use another unit vector $\vec{c}$, so that we can subtract $P(\vec{a},\vec{c})$ from $P(\vec{a},\vec{b})$.

$$P(\vec{a},\vec{b}) - P(\vec{a},\vec{c}) = -\int d\lambda\ \rho(\lambda)\ \Big(\alpha(\vec{a},\lambda)\alpha(\vec{b},\lambda) - \alpha(\vec{a},\lambda)\alpha(\vec{c},\lambda)\Big) \tag{12}$$

$$= -\int d\lambda\ \rho(\lambda)\ \Big(\alpha(\vec{a},\lambda)\alpha(\vec{b},\lambda) - \alpha(\vec{a},\lambda)\alpha^2(\vec{b},\lambda)\alpha(\vec{c},\lambda)\Big) \tag{13}$$

$$= -\int d\lambda\ \rho(\lambda)\ \Big(1 - \alpha(\vec{b},\lambda)\alpha(\vec{c},\lambda)\Big)\ \alpha(\vec{a},\lambda)\alpha(\vec{b},\lambda) \tag{14}$$

Since we know $\alpha(\vec{a},\lambda) = \pm 1$ for any $\vec{a}$ and λ, the second line is simply multiplying one term by unity. Factoring out the common terms from the second line yields the third.

At this time, we should explore some of the inequalities $\alpha(\vec{a},\lambda)$ obeys. First, since its magnitude is always 1 for any arguments, we can write, for all $\vec{a}$ and $\vec{b}$,

$$-1 \le \alpha(\vec{a},\lambda)\alpha(\vec{b},\lambda) \le +1 \tag{15}$$

or alternatively

$$|\alpha(\vec{a},\lambda)\alpha(\vec{b},\lambda)| \le +1\ . \tag{16}$$

Equation 15 also tells us that

$$1 - \alpha(\vec{b},\lambda)\alpha(\vec{c},\lambda) \ge 0\ . \tag{17}$$

Now we can tackle our expression for the difference of $P(\vec{a},\vec{b})$ and $P(\vec{a},\vec{c})$. Taking the absolute value of both sides gives

$$\Big|P(\vec{a},\vec{b}) - P(\vec{a},\vec{c})\Big| = \left|\int d\lambda\ \rho(\lambda)\ \Big(1 - \alpha(\vec{b},\lambda)\alpha(\vec{c},\lambda)\Big)\ \alpha(\vec{a},\lambda)\alpha(\vec{b},\lambda)\right|\ . \tag{18}$$

Applying the triangle inequality, which says that the absolute value of a sum is less than or equal to the sum of the absolute values of the terms, we find

$$\Big|P(\vec{a},\vec{b}) - P(\vec{a},\vec{c})\Big| \le \int d\lambda\ \Big|\rho(\lambda)\ \Big(1 - \alpha(\vec{b},\lambda)\alpha(\vec{c},\lambda)\Big)\Big|\ \cdot\ \Big|\alpha(\vec{a},\lambda)\alpha(\vec{b},\lambda)\Big|\ . \tag{19}$$

Applying inequality 16,

$$\Big|P(\vec{a},\vec{b}) - P(\vec{a},\vec{c})\Big| \le \int d\lambda\ \Big|\rho(\lambda)\ \Big(1 - \alpha(\vec{b},\lambda)\alpha(\vec{c},\lambda)\Big)\Big| \tag{20}$$

$$= \int d\lambda\ \rho(\lambda)\ \Big(1 - \alpha(\vec{b},\lambda)\alpha(\vec{c},\lambda)\Big) \tag{21}$$

$$= 1 - \int d\lambda\ \rho(\lambda)\alpha(\vec{b},\lambda)\alpha(\vec{c},\lambda)\ . \tag{22}$$

We drop the absolute value sign between the first and second lines because both are positive. The final step performed was simply a result of the normalization of $\rho(\lambda)$.

This is the final result, a form of Bell's inequality. We may clarify and write

$$|P(\vec{a},\vec{b}) - P(\vec{a},\vec{c})| \leq 1 + P(\vec{b},\vec{c}) \ . \tag{23}$$

Let us examine further this inequality, which we derived solely from the existence of the hidden variables λ. Suppose $\vec{a}$ is the unit vector along the x-axis and $\vec{b}$ is the unit vector rotated $\frac{\pi}{4}$ from $\vec{a}$. Quantum mechanics predicts that $P(\vec{a},\vec{b})$ would be equal to $-\cos\frac{\pi}{4} = -\frac{1}{\sqrt{2}} \approx -0.707$. If $\vec{c}$ bisects the small angle formed by two other vectors, we find that $P(\vec{a},\vec{c}) = P(\vec{b},\vec{c}) = -\cos\frac{\pi}{8} \approx -0.924$. This yields the startling inequality

$$.217 \leq 0.076 \ , \tag{24}$$

meaning that the quantum mechanical prediction of Eq. 8 violates Bell's inequality.

To show that this is not a special case, let us analyze a more general example. Keeping the setup of $\vec{c}$ bisecting the angle between $\vec{a}$ and $\vec{b}$ (which we call 2θ for convenience), but expressing the dot product in Eq. 8 as the cosine of the angle between $\vec{a}$ and $\vec{b}$, Eq. 23 becomes

$$|-\cos 2\theta + \cos\theta| \leq 1 - \cos\theta \ . \tag{25}$$

Taking the left hand side to be nonnegative, which is easily verifiable for the interval of $\theta \in (-\frac{\pi}{2}, \frac{\pi}{2})$, and using the double angle formula, we find

$$2\cos\theta \leq 1 + \cos 2\theta \tag{26}$$

$$\cos\theta \leq \cos^2\theta \tag{27}$$

$$\cos\theta \geq 1 \ \text{ or } \ \cos\theta \leq 0 \ , \tag{28}$$

neither of which can hold on our domain. We conclude that if quantum mechanics is an accurate description of physical law then Bell's inequality must be violated for not just our specific example of $\theta = \frac{\pi}{8}$ but for a broad class of measurement configurations.

Bell's theorem is a statement of the contradiction between quantum mechanical theory and local hidden variable theories. Bell's theorem says that all local hidden variable theories share this fundamental disagreement with quantum theory.

7 Experiment

We have shown analytically that any local hidden variable theory offers a prediction inconsistent with quantum mechanics. So, which theory's prediction is confirmed?

This is not a matter of philosophy or mathematics, but of measurement. Even though quantum mechanics works well to describe the microscopic world, it could be that it does not work as well as a hidden variable theory does. We cannot simply argue that quantum mechanics is right, or that there is a local hidden variable theory that is right. Indeed, to discover a new, deeper theory, we must strive to find the faults with the currently accepted one.

In 1982, Aspect et al. performed an analogous experiment to the one we have been discussing and found that indeed $P(\vec{a},\vec{b}) = -\vec{a}\cdot\vec{b}$. However, the separation between Alice's sensor's decision to measure a particular $\vec{a}$ and Bob's sensor measuring the B particle along $\vec{b}$ was not spacelike. Instead, detractors argued, some sort of signal could have traveled from Alice's sensor to Bob's sensor in time to influence Bob's sensor's measurement. Thus, critics said, who knows if the sensors were in some way, conspiring to deceive us into believing quantum mechanics?

The Aspect experiment also suffered another flaw. What if the detectors were simply less efficient at detecting one kind of photon than another. Then, the detectors would miss a significant number of the experiments, and we might be convinced of quantum mechanics by a statistical coincidence. Most find this experimental flaw to be much less important than the lack of spacelike separation.

As recently as 1998, Weihs et al. performed an experiment which had strict spacelike separation—there could not possibly be an Alice-and-Bob conspiracy, because Bob could not know Alice's choice to measure along $\vec{a}$. The experiment could still suffer from detector efficiency problems like the Aspect experiment, but having implemented the spacelike separation, most are convinced of Weihs' results even in light of this possible flaw. Weihs performed the same experiment we have been discussing throughout, except that instead of spin-$\frac{1}{2}$ particles, he used photons. From familiarity with electromagnetism, we know that light can have linear polarization and linear polarizations can combine in superposition to give clockwise and counterclockwise polarizations—known as *helicity states.* The crux of the matter is that these helicity states make the photon a two state system, like a spin-$\frac{1}{2}$ particle. Thus, we can create pairs of photons which are in the singlet state shown in Eq. 2.

A source of these spin singlet pairs was located at the origin and sent the photons on their way. While the photons were mid-flight, Alice randomly decided to measure $\vec{a}$ and Bob decided to measure $\vec{b}$, but these decisions and the opposite observations were spacelike separated to prevent a sensor conspiracy. A schematic spacetime diagram of the experiment can be seen in Fig. 2. The random selection of $\vec{a}$ and $\vec{b}$ was by no

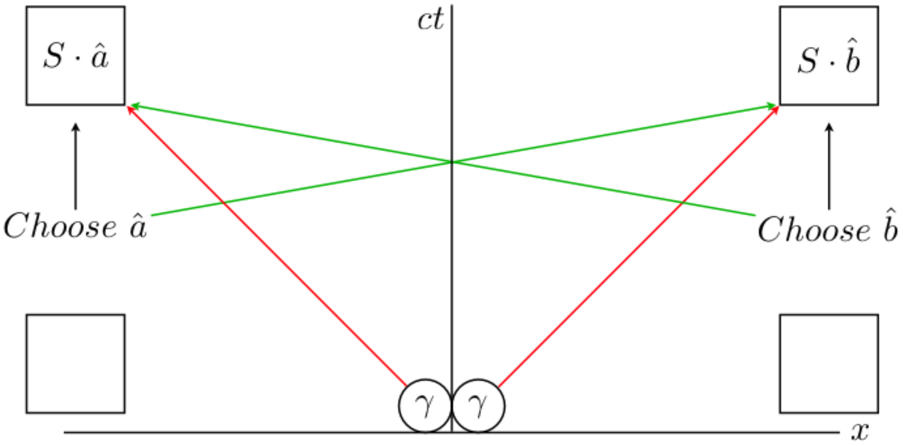

Figure 2: A schematic of the experimental setup used by Weihs to determine $P(\vec{a}, \vec{b})$. Alice is to the left, Bob to the right.

means a trivial task, because Alice and Bob had to make their decisions *very* quickly, since the photons could traverse the whole setup in much less than a second. Alice reported $\vec{a}$ and up (or down) to one computer, Bob reported to a separate computer. At the end of the experiment, the two computers compared lists and found that indeed $P(\vec{a}, \vec{b}) = -\vec{a} \cdot \vec{b}$. Quantum mechanics prevailed.

So, how do these nonlocal correlations jibe with special relativity? Well, we concluded that no communication can cross a spacelike separation, for then we could boost into a frame where the message is received before it is sent. Can Alice convey a message to Bob by the nonlocal correlations in experiments of this kind? She cannot. Alice cannot *choose* the outcome of her measurement—she will measure up half the time and down the rest, at random. If Bob measures along the same axis as Alice, he will measure the opposite of what she measures. However, Bob's list will still seem like a random distribution of ups and downs. Indeed, it will be. Only after the fact, when Bob and Alice compare their lists, do we notice the unusual correlation. Alice cannot convey something to Bob because she cannot decide to measure the particle to be up (or down), and so Bob's list looks just as random as Alice's, conveying no message and preserving causality.

8 Conclusion

Some might call it unfortunate that there cannot be a hidden variable theory which reproduces the results of quantum mechanics because it means that nature is, at its heart, *fundamentally* probabilistic, and this clashes with their philosophical ideas of determinism. However, as any good scientist must, we embrace experiment as the ultimate arbiter of the truth of physical law. The completeness conditions of Einstein, Podolsky, and Rosen, which were asserted as simple, reasonable, and "obviously" true, are wrong. Their ideas that would reduce quantum mechanics to a theory which incorporated probability as a way to make up for its inadequate description of physical reality are now understood to instead point out the truly weird side of nature. Measurements of spins with spacelike separations really can be correlated in a quantum-mechanical way, against all intuition.

Quantum mechanics isn't "missing" anything—it's how the world works at the fundamental level. It is a comforting realization that every laboratory experiment ever performed at the scales where quantum mechanics reigns has agreed with the theory's predictions. To create such an accurate formalism that is incompatible with Bell's inequalities, and to have such a theory succeed, is a testament to human thought and understanding.

Acknowledgements

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