Compton Scattering

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We calibrate the zero-angle of our apparatus and its energy scale for use in Compton a scattering experiment. We measure the electron Compton scattering cross-section via attenuation in $C_{10}H_{11}$ to be $3.2 \pm 0.1 \cdot 10^{23}$ cm^2. By doing a fit to Compton's formula, we find the rest mass of the electron to be 500 ± 120 keV.

1. INTRODUCTION AND STATEMENT OF PROBLEM

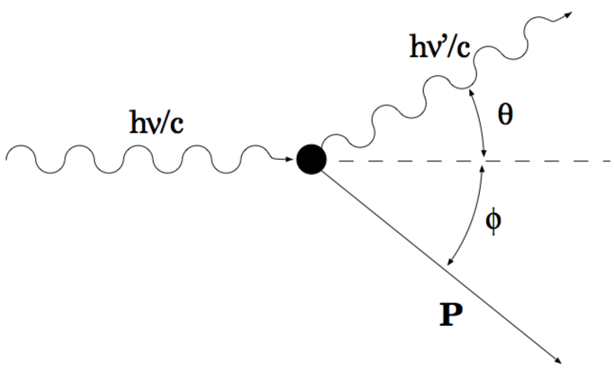

FIG. 1: The physical process which is explained by Compton scattering. The incoming photon scatters off of a particle at an angle θ. To maintain energy and momentum conservation, the frequency of the photon must change. Image taken from [1].

For many years, the nature of light puzzled physicists. Christaan Huygens put forward the first comprehensive theory of light, and described light as a wave. He was able to demonstrate how waves could interfere to form wavefronts and could travel in a straight line. Not much later, Isaac Newton put forward his "corpuscular" theory, in which he described light as particulate. Newton used this model because it made the analysis of reflection intuitive, but could also explain more difficult phenomena, like refraction and the splitting of sunlight into its spectrum by a prism.[2] Because of Newton's intellectual status, his model of light went essentially unchallenged and Huygens' model fell out of use. However, in time, evidence was discovered which supported the wave model, such as characteristic interference patterns and other diffraction phenomena, which could only be explained via superposition, a feature of waves and not of particles. To complete the picture, Maxwell translated Faraday's model of "lines of force" in the ether into his famous equations and was able to directly derive the wave equation from his original equations.[3] So well accepted was the wave theory by 1905, that when Einstein explained the photoelectric effect by explaining that one could view Planck's result, $E = nh\nu$, as n photons, each with energy $h\nu$, he titled his paper "*On a Heuristic Viewpoint Concerning the Production and Transformation of Light*". In the title of his Nobel-winning work, he uses the word "heuristic," implying that it is only useful to think of light in this way, but not that it is actually true.[4] Einstein's explanation of the photoelectric effect helped propel the idea of light's dual wave-particle nature. It is only with the context of the dual nature of light that one can understand Compton scattering. If light were purely a particle, we would be unable to explain diffraction and interference phenomena. If light were purely a wave, we would only observe Thomson scattering, meaning that when it scattered off of an electron, the electron would vibrate at the same frequency as the incoming light, and therefore the emitted light would have the same frequency as well. We can see Thomson scattering, but also observe that the scattered photon can scatter and have a different frequency as the incoming photon.

2. THEORY

Compton scattering can only be understood via an application of both special relativity and the quantum hypothesis. Using special relativity's form of energy conservation, where E is the energy, m the rest mass of the particle in question, p its momentum, and c the speed of light.

$$E^2 = (mc^2)^2 + (pc)^2 \tag{1}$$

and momentum conservation,

$$\boldsymbol{P^2 = p^2 + p'^2 - 2\, p \cdot p'} \; . \tag{2}$$

Adding the quantum hypothesis,

$$p = \frac{h\nu}{c} \; , \tag{3}$$

where h is Planck's constant, and ν is the frequency of the photon, we can derive our final result easily. By combining these three formulae, weconclude that

$$\Delta\lambda = \lambda_c \left(1 - \cos\theta\right) \tag{4}$$

where λ_c, the Compton wavelength, is equal to $\frac{h}{mc}$.

From this formula, it is clear that the greatest $\Delta\lambda$ can be is $2\lambda_c$, which corresponds to the photon scattering in the direction from which it came. This corresponds to classical elastic collisions, where a particle would relinquish the most energy it can to its target if it heads back where it came from after the collision.

3. EXPERIMENTAL SETUP

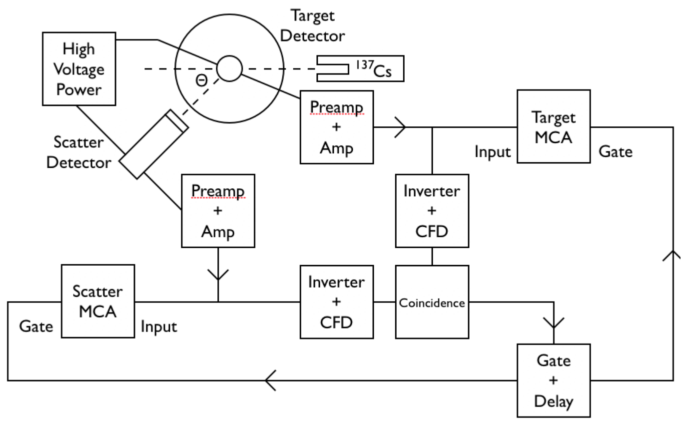

FIG. 2: This is a schematic of the main apparatus and setup of our experiment. Θ can be varied in order to explore the relationship between the scattering angle and the energy.

In order to measure the angular dependence of the photon's change in energy, we constructed an apparatus which would allow us to change the position of a NaI scintillator relative to a source of photons. Because we would like to be able to measure a change in wavelength or energy, it is vital that we know the energy of the photons striking our target. We used a ^{137}Cs source which decays and gives off a 661.6 keV photon, through a metastable Barium configuration, as follows:

$$^{137}Cs \rightarrow {}^{137m}Ba + e^- + \nu_e$$
$$^{137m}Ba \rightarrow {}^{137}Ba + \gamma$$

This decay allowed us to fix an energy on the photons that were escaping the howitzer source and heading for the target.

The scatter scintillator was set at various angles in order to capture the relationship between scatter angle and the energy of the scattered photon. This allows us to pin down two of the three energy values in the collision, the third being the energy of the electron. If, we use another scintillator as our target, then we will be able to measure the electron's energy as well. However, in order to measure the correct electron energy, some additional circuitry is necessary. We added a coincidence component, which allows us to select only the signals from the target detector which occur close to the signals from the scatter detector. Using the coincidence component prevents the MCA from reading input data unless the other both detectors have produced a signal recently, meaning that we can pick out signals from both detectors that correspond to the same scattering event.

It is advantageous to add the complication of measuring the electron's energy because by measuring both the scattered photon's energy and the electron's energy we effectively measure the total energy after the scattering event. We can compare the energy before and after the event, and if they consistently match, feel reasurred that we know the true mechanism of the scatter. Without measuring the electron's energy, we could still measure the dependence of the scattered photon's energy on the scattering angle, but could doubt the process by which the photon scatters.

4. ENERGY CALIBRATION

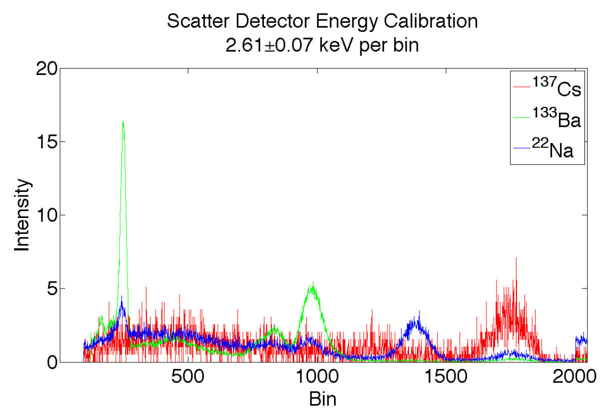

FIG. 3: These spectra were measured without coincidence for a brief amount of time. In both the ^{133}Ba and ^{22}Na spectra, there is a small cesium peak. They remain because the lead door that closed off the cesium howitzer did not catch all of the radiation. This relates to the attenuation of the photons, which is discussed in section 6.

In order to calibrate our energy scale, we took simultaneous data on both detectors for three samples. We measured ^{22}Na which undergoes β^+ decay, and therefore gives off a 511 keV line. This line can be seen in Figure 3 at approximately bin 1400. Additionally, we measured ^{133}Ba which gives off characteristic photons at 81 keV, 302 keV, and 356 keV.[1] These lines can be seen at bins 250, 800, and 1000, respectively. As an additional point for calibration, we measured the 661.6 keV ^{137}Cs peak as well, which shows up near bin 1750.

The uncertainty on each peak's position was calculated by measuring the full-width at half-max and dividing by 2.354, as suggested in section 2.3 of Bevington.[5] Once the values and their uncertainties were determined, a linear fit was performed in order to determine the scale of the data.

To align the scale of the MCAs, the gains on the amplifiers were adjusted during the setup of the experiment while barium decayed between the two detectors. By observing barium, we could visually confirm that three points on the scale roughly aligned. The results of calculating the scales for the two MCAs yield an energy calibration of 2.61 ± 0.07 keV per bin and 2.59 ± 0.07 keV per bin for the scatter and target MCAs, respectively.

5. ANGULAR CALIBRATION

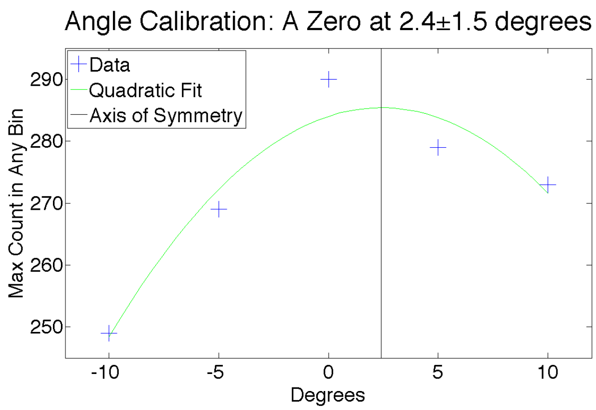

FIG. 4: The uncertainties on all of the data points were negligible and equal, so the quadratic fit was peformed on the data without errors using Matlab's polyfit function. Note that the count at -5 and 10 degrees are approximately equal, meaning that the zero should be near their average, 2.5 degrees.

As well as calibrating the energy, we must also calibrate the angle if we are to be confident in our angle dependence. To calibrate the angle, we would like to determine what reading of the angle is the phenomenon symmetric about, and set this angle to zero, so that the measurements are symmetric about the zero, as is suggested by (4). We setup the experiment and turned the coincidence requirement off in order to get a larger, more statistically significant count, since this calibration only depends on the scatter detector, and not on the target detector. We took counts for 60 live seconds at 0°, ±5°, and ±10°.

The data points were calculated for each degree trial using the following procedure. First, find the maximum count in any bin. Then, find all the bins whose count is greater than half of that maximum. If we average these bin numbers together, we should be approximately in the center of the peak, meaning the count in that bin is a good estimate for the count at the peak value.

Using these data, a quadratic fit was performed with Matlab's polyfit function. By finding the axis of symmetry of the resulting parabola, we have estimated the angle about which the scattering is symmetric. By this method, we find that the zero angle is roughly 2.5±°. As a validation of this approximation, we note that the counts at -5° and 10° are approximately equal, and hence the axis of symmetry should be close to their average.

6. ATTENUATION

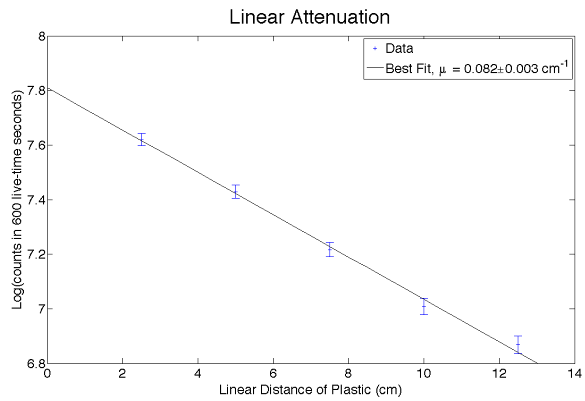

FIG. 5: This linear relationship is visually obvious and can be fitted very well. Since the

When a signal moves through matter, its amplitude and intensity decrease. Similarly, when the beam of photons moves through matter, inevitably the number of photons that pass through decreases. This reduction is known as attenuation. Attenuation has a larger effect when you travel through more matter, meaning that if two identical beams went through different amounts of the same material, the one which went through the greater linear distance would have a lower intensity on the other side.

The intensity of a collimated beam, such as the γ-rays from the howitzer, after a traveling through a linear distance x of material, is given by

$$I(x) = I_0 e^{-\mu\ x} \tag{5}$$

where I_0 is the original intensity of the beam, and μ is the linear attenuation coefficient.[1]

We performed measurements with the scatter detector across from the howitzer. Immediately in front of the howitzer we placed blocks of plastic, polyvinyltoluene ($C_{10}H_{11}$), and took long data for 10 minutes each trial. We ran one trial for no blocks of plastic, and an additional trial for one, two, three, four, and five plastic blocks, each of which were 2.5 cm thick. By plotting the log of the counts against the linear distance through which the beam traveled, we can determine μ, by calculating the additive inverse of the slope. When this is done, we find that $\mu_{C_{10}H_{11}} = 0.082 \pm 0.003$ cm^{-1}.

Additionally, we can determine the Compton scattering cross section, σ_{total}, of an individual electron, which represents how large a target an electron is for a photon for this process. We know that

$$\sigma_{total} = \frac{\mu}{n_e} \tag{6}$$

where n_e is the electron number density for the material. n_e is easily calculated from N_A, the density of $C_{10}H_{11}$, the number of proton in C and H, and the number of neutrons in C and H, and is approximately $2.57 \cdot 10^{21}$ cm^{-3}, yielding a Compton cross section of $(3.2 \pm 0.1) \cdot 10^{-23}$ cm^2. Published values range between 10^{-21} and 10^{-25} cm^2.

7. COMPTON SCATTERING

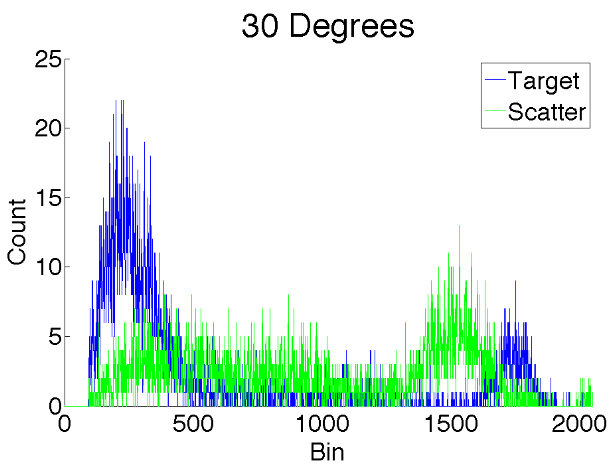

FIG. 6: Well defined peaks, 661.6 keV Cesium peak. The 661.6 peak is visible in the target spectrum because at all times photons are simply being absorbed into the scintillator instead of scattering.

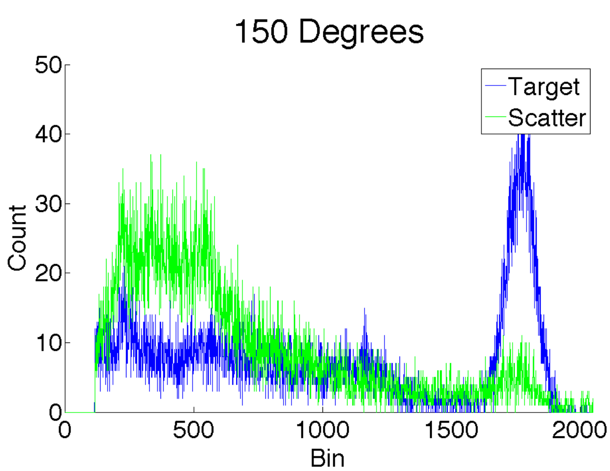

FIG. 7: In this spectrum it is more difficult to see well defined peaks. Well defined peaks, 661.6 keV Cesium peak. A small 661.6 keV peak is visible in the scatter spectrum because of Thomson scattering, as discussed in Section 1.

In an attempt to measure the Compton effect, we probed 30°, 60°, 90°, 120°, and 150°. The trials for the acute angles lasted 300 live-time seconds. The 90°trial lasted 1600 seconds, the 120°trial 2400 seconds, and the 150°trial approximately 3600 seconds. The longer trial times for the greater angles were an attempt to prevent noise from dominating the spectra and we believed would provide us with clear, well-defined peaks with small error. Instead, these spectra were significantly worse than the acute angles, which ran for comparatively brief trials, with errors on the scatter peak being up to two and six times as large, as can be seen in Table I. Since the general shape of these spectra is correct and it is simply too noisy to determine the peak with certainty, these large errors probably arise from the noise in the scintillators. It is possible that our discriminator were not set to a high enough energy or that there was a source of noise we did not otherwise consider.

TABLE I: A table of summarized data of energy dependence on angle.

Angle (degrees)[a]	Target Peak	Scatter Peak	Sum[b]
30	230±90	1500±100	1700±135
60	610±60	1170±30	1780±70
90	1290±40	400±160	1700±170
120	1270±50	380±160	1650±170
150	1170±80	390±190	1550±210

[a] When doing the linear fit, these angles were adjusted according to the calibration, as discussed in section 5.

[b] The sums and their errors ignore adjustments for different energy scales, as those systematic differences are insignificant when compared to the errors from the peak data.

As can be seen in 4, there should be a linear relationship between λ or $\frac{1}{E_\gamma}$ and $1 - \cos\theta$, with slopes being λ_c and $\frac{1}{m_e c^2}$ respectively. When the inverse of the bin is fitted linearly against the trigonometric expression, a slope of $.57 \pm .13 \cdot 10^{-3}$ is obtained. If fit only the acute (less noisy) angles, a slope of $.52 \pm .13 \cdot 10^{-3}$ is obtained. When mapped onto an energy scale, these slopes then correspond to electron masses of 450 ± 100 keV and 500 ± 120 keV, respectively. While the error on these values is large (approximately 20%), both values contain the accepted electron mass of 511 keV. Due to the small error on the acute angles and the large error on the greater angles, the fit with all the data included is dominated by the acute points, and the fit line only hits one of the error bars of the greater angles. Thus, we cannot conclusively state that Compton's formula holds for large angles, but can confirm it for small angles.

8. CONCLUSIONS

We were successfully able to calibrate our energy scale as well as our angle. Additionally, our attenuation data conformed to the decaying exponential relationship extremely well, as shown in Figure 5. Through this measurement, we were able to calculate the mass attenuation coefficient of $C_{10}H_{11}$ and the Compton scattering cross section of an electron to be 0.082 ± 0.003 cm^{-1} and

$(3.2 \pm 0.1) \cdot 10^{-23}$ cm^2 respectively.

Due to our extremely large error in our data for the greater scattering angles, we are unable to definitively accept Compton's formula as governing the scattering phenomenon at large angles. As an alternative to spending such a long amount of time on larger angles, in the future it would be more effective to measure the correlation for smaller angles, as those seem to have a smaller error, and have plots whose peaks are obvious instead of broad and noisy. In addition, raising the cutoff levels for the discriminators might decrease the possibility of triggering on noise and allowing the noise to pass through the coincidence component.

[1] MIT Department of Physics, *Compton Scattering*, JL-Exp01.pdf (2006).

[2] W. Authors, *Wave-Particle Duality*.

[3] D. Kaiser, *Einstein, Oppenheimer, and Feynman: Phsyics in the 20th Century - Physics Then and Now* (2006).

[4] D. Kaiser, *Einstein, Oppenheimer, and Feynman: Phsyics in the 20th Century - Rethinking Light* (2006).

[5] P. Bevington and D. Robinson, *Data Reduction and Error Analysis for the Physical Sciences* (McGraw-Hill, 2003).