

The Speed and Mean Lifetime of Cosmic Ray Muons

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We measure properties of cosmic-ray muons which reach sea level. Using two paddle scintillators we measure their average velocity to be $1.02c \pm 0.09c$, demonstrating these particles' high energy. Using a cylinder of plastic scintillator we capture muons and measure their mean lifetime, τ , to be 2.26 ± 0.07 . Through the electroweak theory, we extrapolate this to mean that the mass of the muon is $105.2 \pm 0.6 \text{ MeV}$. Because the mean lifetime is much shorter than the required flight time to the surface from their creation point in the upper atmosphere, we validate relativistic time dilation.

1. PURPOSE

Cosmic rays have been used as a source of high-energy particles since their discovery in the 1930s. Cosmic rays consist of primarily of protons with a small portion of helium nuclei and an even smaller portion of heavy nuclei. When these nuclei collide with particles in the upper atmosphere (approximately 15 km), the collision is so energetic that the ray comes apart, producing a shower of other particles.

Going at very close to the speed of light, c , the 15km trip would take $50\mu\text{s}$. Muons, the particles which most frequently reach Earth's surface, have a mean-life, τ , of $2.2\mu\text{s}$. Now have a dilemma: with these two figures and a classical interpretation, we would expect to observe about 1 muon for every 10^{10} muons created.

One particle in ten billion is hardly a measurable portion, and we seem to measure on the order of one muon every second per square centimeter. With other techniques, including high-altitude balloon measurements, we know that the muon production rate is not nearly ten billion per square centimeter per second. Instead, we must find an alternate explanation of the seemingly high sea-level muon flux.

By measuring the muon's high speed and short mean lifetime, we provide evidence for the phenomenon of relativistic time dilation. The muons, traveling a speeds very close to c experience less time than we at rest measure, allowing them a reasonable chance of reaching Earth's surface.

2. SPECIAL RELATIVITY

Special Relativity, proposed by Albert Einstein in 1905, holds two postulates to be absolute truths:

1. The laws by which the states of physical systems undergo change are not affected, whether these changes of state be referred to the one or the other of two systems of co-ordinates in uniform translatory motion
2. Any ray of light moves in the "stationary" system of co-ordinates with the determined velocity c , whether the ray be emitted by a stationary or moving body...[1]

In his famous paper *On The Electrodynamics of Moving Bodies*, Einstein uses these postulates to rederive the coordinate transformations already noticed by Lorentz, giving physical understanding to what was previously simply a mathematical curiosity.[2] These coordinate transformations leave Maxwell's equations unchanged, bringing our understanding of electromagnetism into compliance with postulate 1, and can be summarized as

$$\begin{pmatrix} x' \\ y' \\ z' \\ ct' \end{pmatrix} = \begin{pmatrix} \gamma & 0 & 0 & -\beta\gamma \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\beta\gamma & 0 & 0 & \gamma \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ ct \end{pmatrix}$$

where all of primed variables are measured in an inertial frame of reference which moves at speed βc with respect to the unprimed frame along the axis of motion, which we choose for convenience to be the x -axis. γ , the Lorentz factor, is defined to be $\frac{1}{\sqrt{1-\beta^2}}$. At speeds with which we are familiar through first-hand experience, β is so close to 0, that γ is effectively 1, reducing these rules for relativistic transformations to their Galilean counterparts.

3. COSMIC RAY MUONS

Cosmic rays constitute a relativistic gas of mostly protons which have been accelerated by galactic electromagnetic fields. As the protons near Earth, they hit nitrogen and oxygen nuclei in the upper atmosphere (at approximately 15km). This collision produces positive π -mesons the corresponding antiparticle π^- , which are both called pions.[3] Pions decay quickly into muons via the decay

$$\pi^+ \rightarrow \mu^+ + \nu_\mu \quad (1)$$

with a corresponding decay for the π^- particle.

The resulting muons decays by the process

$$\mu^+ \rightarrow e^+ + \nu_\mu + \bar{\nu}_e \quad (2)$$

with a corresponding decay for the μ^- . If we suppose that we have a muon at time t then the odds the muon survives dt is $r \cdot dt$, where r is a characteristic decay rate. Then, if the probability $p(t)$ describes the chance

a particle which exists at time $t = 0$ exists at time t , by compounding the probability $p(t + dt) = p(t)(1 - r dt)$, we can show that $p(t) = e^{-rt}$. We define τ to be the value for which the probability is $\frac{1}{e}$, and rewrite $p(t)$ as $e^{-t/\tau}$.

Muons were the first elementary particles found to decay into other elementary particles, and even though their lifetime is only a few microseconds, this is much longer than any other particles with which we do not regularly interact.

4. RELEVANT STATISTICS

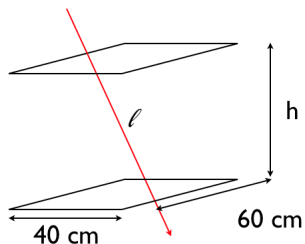


FIG. 1: Scintillator paddle geometry. The red vector represents one path a muon might take that our apparatus would detect.

In order to determine the speed of the muons, we will measure the time they take to traverse the distance between two scintillator paddles. Intuitively, we assign the particles an average velocity based on the average distance traveled divided by the average time it takes to traverse the distance. With the paddle geometry shown in Figure 1, calculating the mean distance traveled is a nontrivial task.

We let x_1 and x_2 correspond to the short dimension of the top and bottom paddles respectively, y_1 and y_2 to the short dimension of the two paddles, and ϕ to the azimuthal angle of the flight path with respect to the surface. We can vary h , the height between the two paddles. We define l to be the distance traveled by the muon, and note that it is given by $l = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + h^2}$.

Careful measurements can show that the direction of muons at sea level follow a $\cos^2 \phi$ distribution.[4] By adapting the familiar mean-value formula

$$\langle f \rangle = \frac{\int f p(f) df}{\int p(f) df} \quad (3)$$

to our geometry, we find that the mean distance traveled given a paddle separation h is given by

$$\langle l \rangle = \frac{\int_0^{60} dy_2 \int_0^{60} dy_1 \int_0^{40} dx_2 \int_0^{40} dx_1 l \cos^2 \phi}{\int_0^{60} dy_2 \int_0^{60} dy_1 \int_0^{40} dx_2 \int_0^{40} dx_1 \cos^2 \phi} \quad (4)$$

These integrals must be evaluated numerically, as even the symbolic math program Mathematica by Wolfram

Research cannot express the integral in closed form, as a function of h . However, we can nonetheless reason through the behavior of $\langle l \rangle$ as a function of h .

When h is large, the second paddle subtends a small solid angle from the point of view of the first paddle. Therefore, the azimuthal angle will have a very small spread, leading us to conclude that $\langle l \rangle$ will be very close to h . Analogously, when h is small, the second paddle subtends a very large solid angle, creating more paths which are much longer than h . Accordingly, we expect that for small h , $\langle l \rangle$ will differ significantly from it.

5. FLIGHT SPEED

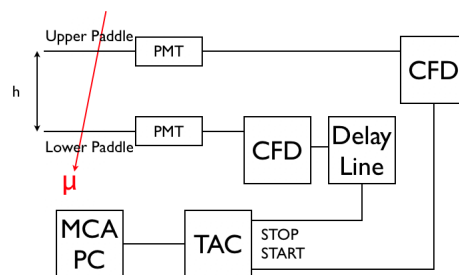


FIG. 2: Diagram of the flight time apparatus. The bottom paddle is on an adjustable track, so that we can vary h between samples. Though not obvious in the diagram, an equal amount of cable is between both PMTs and their respective CFDs.

The experimental apparatus shown in Figure 2 is very elementary. There are two scintillator paddles, the top fixed and the bottom on a sliding track so that we may adjust their separation. When a muon passes through the paddles and, the scintillations are picked up by photomultiplier tubes and the signals are transmitted to constant fraction discriminators (CFDs), after which the signal from the bottom paddle is delayed. The signals are fed into a time to amplitude converter, which is then sent into the multichannel analyzer (MCA) and PC combination. This allows us to take spectra which are dependent on the separation h . With a smaller h a spectrum will be closer to the origin than a spectrum with a larger h because it takes more time to traverse a larger distance than a smaller one. We can calibrate our MCA with a time calibrator, yielding a correlation between MCA bin and time passed.

To experimentally determine the average speed of the muons, we first set the paddles close together and take a brief sample. We then set the paddles farther apart and take a longer sample to balance the reduced flux and obtain a similar count. Once these two spectra are taken, we can find the average velocity, $\langle v \rangle$, using

the difference formula

$$\langle v \rangle = \frac{\langle l_{long} \rangle - \langle l_{short} \rangle}{\langle t_{long} \rangle - \langle t_{short} \rangle} \quad (5)$$

As shown in Figure 3, the spectra are not precisely de-

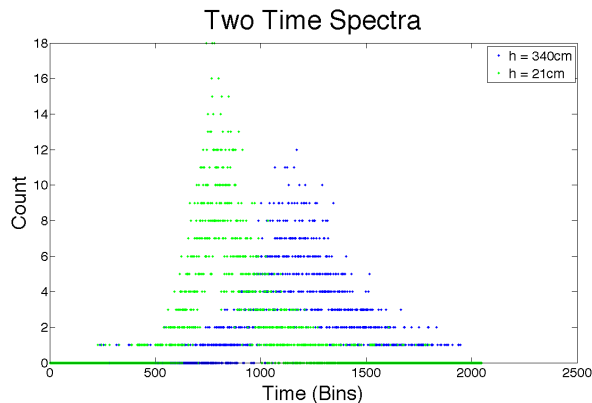


FIG. 3: Two spectra from different heights.

fined. This is expected, because muons reach the surface with varying energies and at varying angles, meaning that they do not all take the same time to cross the paddles.

The two shown spectra shown are $h=21\text{cm}$ and $h=340\text{cm}$. The difference in average flight paths for these two separations is 313.1cm , which was by the difference in their $\langle l \rangle$, found by evaluating (4). Since the heights were measured with little error and $\langle l \rangle$ is simply a computation, we assume the error on the difference to be negligible. By finding the mean time of those datapoints within the FWHM of the peak, we find the time difference to be 377 ± 41 bins, or $10.0 \pm 1.1\text{ns}$.

These data lead us to believe that the muons were, on average, traveling $1.02c \pm .09c$. Though this figure is greater than the speed of light, it easily leaves open the possibility of traveling between c and $.93c$ as these proper relativistic speeds are within one σ of the average value. We will assume the average speed is $.99c$ as an arbitrary choice in order to show the value of time dilation.

6. MEAN LIFETIME

The experimental apparatus for measuring the mean life of the muons, shown in Figure 4, contains much of the same circuitry as the apparatus shown in Figure 2. We have a light tight box, whose interior is entirely dark. In the box is a large scintillator. Muons which penetrate the shielding can be slowed by the scintillator, leaving an path of ionization behind them. This causes a pulse of light in the scintillator, which is picked up by the photo-multiplier tubes (PMTs). If both PMTs observe a large enough pulse, they pass through the CFDs and through the coincidence, which is imposed to eliminate noise. The coincidence sends an undelayed pulse to stop the timer if it had been started incorrectly. The delayed pulse starts

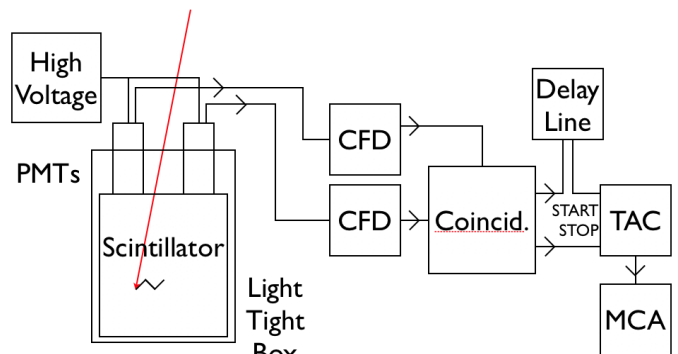


FIG. 4: Diagram of the mean lifetime Apparatus.

the timer. Then, the pulse from the muon’s decay follows the same path. Now the signal halts the previously started timer and begins the timer anew.

Fortunately, over a trial lasting 53600 seconds, we counted only 15331 muons, meaning that on average the time between them was just about three and a half seconds. We do not detect these times because we set the TAC’s range to be sensitive to the microsecond scale, and thus we do not see the times between the muons. It is not detrimental to make the stop signal undelayed because the lifetime obeys exponential decay, which appears the same even under a shift in the “zero” time.

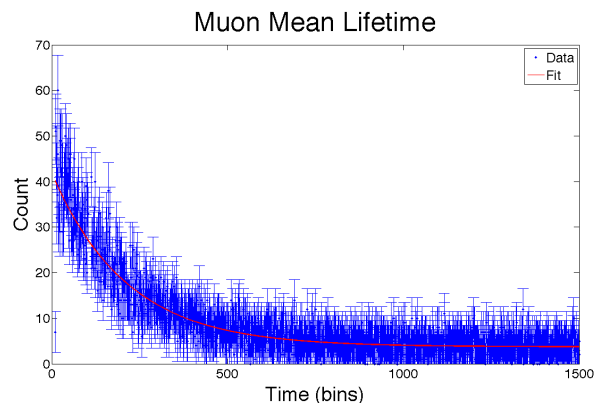


FIG. 5: This sample was run overnight, for a total time of 53600 seconds (a little less than 15 hours). The fit was generated with the `fittemplate` package and a function which computes points of the form (6).[5]

TABLE I: A table summarizing the fitting parameters for the exponential fit.

a	τ (bins)	τ (μs)	b	χ^2_ν
38.0 ± 0.5	211.5 ± 2	2.26 ± 0.07	2.74 ± 0.05	1.22

The data gathered, shown in Figure 5, decays exponentially, as we expect. We used a nonlinear fitting algorithm

to fit the data to a function of the form

$$y(t) = a e^{-t/\tau} + b \quad (6)$$

where a simply determines the vertical scale of the decay, b represents a constant level of noise, and τ is the mean life of the muon. We fit all the bins with at least one count, and set the error on that count to its square-root. We omit bins with a zero count because that would give no error, and completely dominate the fitting quality and distort the result significantly. The results of the fit are given in Table I.

Using this process, we determine the decay time of the muon to be $2.26 \pm 0.07 \mu\text{s}$, which is in good agreement with the accepted value of $2.2 \mu\text{s}$.

Additionally, we can determine the mass of the muon by a formula derived from electroweak theory. Electroweak theory tells us

$$\tau = \frac{192\pi^3 \hbar^7}{G_F^2 m_\mu^5 c^4} \quad (7)$$

where τ is the mean life of the muon, \hbar is Planck's reduced constant, G_F is an electroweak coupling constant, m_μ is the mass of the muon, and c is the speed of light.[6]

When we resolve (7) for m_μ , we get a power law dependence on τ , with exponent of $-\frac{1}{5}$. This works in our favor, helping to reduce the error we measure on the mass of the muon. Using our τ , we calculate the mass of the muon to be $105.2 \pm 0.6 \text{MeV}$, which agrees well with the accepted mass of 105.6MeV .

7. ERROR

The small error and good agreement with accepted values for the mean lifetime experiment show that the sources of error were small. Using a long sample time meant that the distribution came out smoothly, though in order to do the fit we had to remove a stray peak which was centered on bin 100 ($1.06 \mu\text{s}$) and was 10 bins wide. The source of this peak is unknown, but could arise from high-energy photons which successfully penetrated the shielding or other cosmic-ray decay products managing to penetrate the shielding as well. Without an energy calibration for the MCA, we cannot state the energy of this peak.

Another source of error in the mean lifetime experiment was accidental coincidence. If two muons cause pulses within a short time of one another, they will be picked up in the same TAC range, and cause noise. Similarly, muons which are not captured but pass through the light tight box can still cause some ionization, creating stray pulses which are unwanted.

Error on the velocity of the muons come from a variety of sources. One source is that the muons do not

have a singular speed. Instead, they arrive with a variety of speeds. Additionally, the distance traveled is not uniform. As discussed in Section 4, the distance traveled takes on a spread which depends on the distance that separates the paddles. This effect would be smaller for smaller paddles, but we balance our desire to minimize this error with our desire to capture a significant flux of muons.

Another source of error is the size of the paddles. If a muon traveling from the edge farther from the PMT of the top paddle to the edge nearer the bottom paddle's PMT, the start signal will be delayed and the time it takes to travel the path will be underestimated. If we reverse the edges, the stop signal will be delayed and the time overestimated. Again, we must balance this error with a significant flux of muons.

8. CONCLUSIONS - THE AFFIRMATION OF RELATIVISTIC TIME DILATION

While the average speed we measured for the muons was larger than c , we can still find support for the theory of special relativity in our data. Specifically, the phenomenon of time dilation. If, in the laboratory frame, the muons are traveling with $\beta=0.99$, it will take them approximately $50 \mu\text{s}$ in the lab frame to arrive at the surface from their creation point at 15km above sea level. Classically, the time that the muons would experience during this journey would be equivalent, $50 \mu\text{s}$, meaning that the probability that the muon had not decayed by the time its journey would end would be $e^{-\frac{50 \mu\text{s}}{\tau}}$ which evaluates to approximately 10^{-10} . Relativistically, the form would be the same, but instead of $50 \mu\text{s}$, this time would be reduced by a factor of γ , the Lorentz factor. For $\beta=0.99c$, $\gamma = 7.09$, meaning the time experienced by a muon would be very close to $7 \mu\text{s}$, much less than $50 \mu\text{s}$, resulting in an increased probability, roughly 0.04. So, classically we expect to see 1 muon in 10^{10} produced and relativistically we expect to see 4 for every 100 produced.

By measuring muon flux at various altitudes, including upon high mountains and via balloon experiments, we can measure that 4 muons of every 100 is much closer to the actual muon production rate than the classical rate. We are compelled to acknowledge the power of special relativity in explaining why we are able to measure any muons at the surface, given their production rate in the upper atmosphere.

We have measured the mean life of the muon to be 2.26 ± 0.07 , the mass of the muon to be $105.2 \pm 0.6 \text{MeV}$, both in good agreement with their respective accepted values. We have measured the average muon velocity to be $1.02c \pm 0.09c$. Combining these data allow us to provide strong experimental evidence for the effect of relativistic time dilation.

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 - [2] D. Kaiser, *Einstein, Oppenheimer, and Feynman: Physics in the 20th Century - Einstein's Relativity* (2006).
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 - [5] S. Sewell, *Matlab scripts and curve fitting package* (2006), URL <http://web.mit.edu/8.13/matlab/>.
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