PHYS 161	**QUIZ 3**
February 23, 2009	**10 POINTS**

Vectors $\vec{A}$ and $\vec{B}$ have equal magnitude: $5.00m$.

(i) If the sum of the two vectors is $6.00\widehat{\mathbf{j}}m$, determine the angle between the two vectors.

Call the sum $\vec{S} = \vec{A} + \vec{B}$. We are told that $|S| = 6m$ (and also $\vec{S}\cdot\vec{S} = 36m^2$).

Since $\vec{A}$ and $\vec{B}$ have magnitude of $5.00m$, $|A| = |B| = 5m$ and $\vec{A}\cdot\vec{A} = \vec{B}\cdot\vec{B} = 25m^2$,

Take the dot product of $\vec{S}$ with itself:

$$\vec{S}\cdot\vec{S} = \left(\vec{A}+\vec{B}\right)\cdot\left(\vec{A}+\vec{B}\right) \tag{1.1}$$

$$= \vec{A}\cdot\vec{A} + 2\vec{A}\cdot\vec{B} + \vec{B}\cdot\vec{B} \tag{1.2}$$

$$= \vec{A}\cdot\vec{A} + 2|A||B|\cos\theta + \vec{B}\cdot\vec{B} \tag{1.3}$$

Solving, we find

$$\cos\theta = \frac{\vec{S}\cdot\vec{S} - \vec{A}\cdot\vec{A} - \vec{B}\cdot\vec{B}}{2|A||B|} \tag{1.4}$$

Plugging in, we find

$$\cos\theta = \frac{36m^2 - 25m^2 - 25m^2}{2\cdot 5m\cdot 5m} = -\frac{7}{25} \tag{1.5}$$

Then, we find

$$\theta = 1.85459 \text{ radians} = 106.26^\circ \tag{1.6}$$

(ii) What is the length of the vector $\vec{A} - \vec{B}$

Denote the difference by $\vec{D} = \vec{A} - \vec{B}$. We want to find the length of the $\vec{D}$, which is equal to $\sqrt{\vec{D}\cdot\vec{D}}$.

$$\vec{D}\cdot\vec{D} = \left(\vec{A}-\vec{B}\right)\left(\vec{A}-\vec{B}\right) \tag{1.7}$$

$$= \vec{A}\cdot\vec{A} - 2\vec{A}\cdot\vec{B} + \vec{B}\cdot\vec{B} \tag{1.8}$$

$$= \vec{A}\cdot\vec{A} - 2|A||B|\cos\theta + \vec{B}\cdot\vec{B} \tag{1.9}$$

We can now evaluate this. Using our value for $\cos\theta$ found in (1.5), and the magnitudes of $\vec{A}$ and $\vec{B}$ that we know,

$$\vec{D}\cdot\vec{D} = 25m^2 - 2\cdot 5m\cdot 5m\cdot\left(-\frac{7}{25}\right) + 25m^2 \tag{1.10}$$

$$= 64m^2 \tag{1.11}$$

Taking the square-root of both sides, we find

$$|D| = 8m \tag{1.12}$$

□