PHYS 161 March 28, 2009	**QUIZ 6** **10 POINTS**

A block of mass 2.00 kg is resting on the left edge of a larger mass of 8.00 kg. The larger block is resting on a frictionless surface and the coefficient of friction between the two blocks is 0.300. A constant horizontal force of 10.0 N is applied to the smaller block setting it in motion as shown. The length L of the leading edge of the smaller block travels is 3.00 m.

(i) [2+2] Draw the free body diagrams of both the blocks

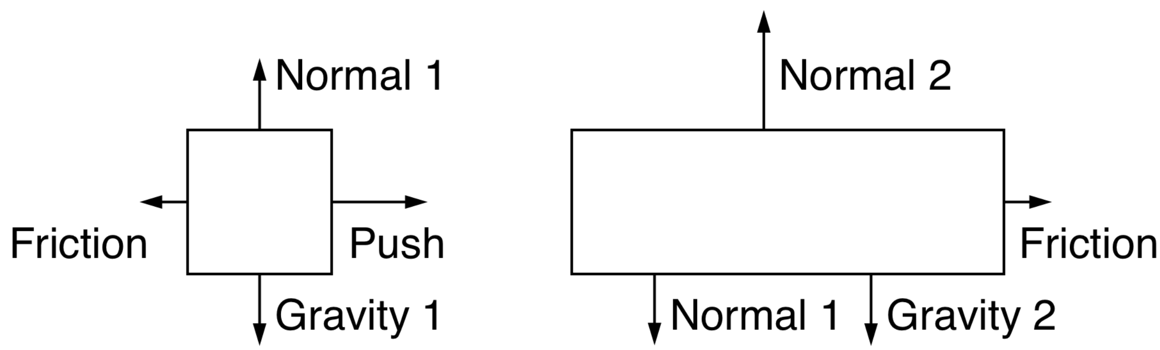

Figure 1: The various forces on the different blocks (arrows are not to scale).

The push is the constant horizontal force. The friction on the smaller block must oppose that, and so points left. The equal-but-opposite frictional force on the larger block must therefore point right. The normal force on block 1 (the top block) must be equal to the force of gravity on block 1 because the block is not accelerating vertically. Thus, the size of the normal force on 1 must be $m_1 g$. The normal force on 2 must prevent block 2 from accelerating vertically, so it must be equal to the size of the normal force on 1 plus the force of gravity on 2.

(ii) [2] How much force is pushing the larger mass?

It should be clear that all of the vertical forces cancel out, because the block will not accelerate upward or downward. Thus, the only remaining force on the larger mass is the friction force from the contact with the lighter mass. What is that force? It is given simply as

$$f = \mu N = \mu(m_1 g) \tag{1.1}$$

Plugging in the numbers gives

$$\boxed{f = 5.88N \approx 6N} \tag{1.2}$$

You get 6N if you choose $g = 10\ m/s^2$.

(iii) [2] In one second, how far has the larger block travelled?

Since the block itself is on a frictionless surface, the net horizontal force is equal to the force of friction. Via $F = ma$, the acceleration of the larger mass must be

$$a_2 = \frac{F}{m_2} = \frac{\mu m_1 g}{m_2} = \mu g \frac{m_1}{m_2} \tag{1.3}$$

which clearly has the same units as g, so it has units of acceleration, which is what we hope. We know that the block started from rest. Thus, it must be that the block obeys the following kinematical equation:

$$x_2 = \frac{1}{2} a_2 t^2 = \frac{1}{2} \left(\mu g \frac{m_1}{m_2} \right) t^2 \tag{1.4}$$

Setting $t = 1s$ gives

$$\boxed{x_2 = 0.3675m \approx 0.375m} \tag{1.5}$$

where you get the second answer for $g = 10\ m/s^2$.

(iv) [2] In the same time, how much has the smaller block travelled?

The total horizontal force on the smaller block is equal to the push P (which is 10 N to the right) and the friction (which is μN to the left). Symbolically,

$$F = P - \mu m_1 g \tag{1.6}$$

The vertical forces cancel, because the block shouldn't move up or down. So we only need to worry about horizontal motion. The horizontal acceleration is simply the horizontal force divided by the mass (via $F = ma$). So,

$$a_1 = \frac{F}{m_1} = \frac{P - \mu m_1 g}{m_1} = \frac{P}{m_1} - \mu g \tag{1.7}$$

Since the block starts from rest, it must also obey the kinematical equation

$$x_1 = \frac{1}{2} a_1 t^2 = \frac{1}{2} \left(\frac{P}{m_1} - \mu g \right) t^2 \tag{1.8}$$

Setting $t = 1s$ gives

$$\boxed{x_1 = 1.03m \approx 1m} \tag{1.9}$$

where the latter is found by $g = 10\ m/s^2$.

It is an interesting feature of (1.8) that it appears that x_1 might be negative if what is inside the parentheses is negative. Should we be worried? We need to be concerned if

$$\frac{P}{m_1} - \mu g < 0 \tag{1.10}$$

Rearranging, we see that the concern arises if

$$P < \mu m_1 g = \mu N = f \tag{1.11}$$

This situation is excluded physically: the friction can never exert more force than the push on the object. The most extreme case of this should be obvious: if we don't push at all (ie. $P = 0$) then the friction should be zero, or things that were sitting still would start moving! That would be strange indeed.

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