PHYS 161	**QUIZ 8**
April 25, 2009	**10 POINTS**

A bullet of mass m and speed v passes completely through a pendulum bob of mass M and emerges with speed $v/2$. The pendulum bob is suspended by a stiff rod of length L. The rod has negligible mass. Express everything in terms of symbols. Use g to denote the acceleration due to gravity.

(i) [2] What is the speed of the bob after the bullet has passed through it?

We cannot conserve energy, because the collision may be inelastic (energy might go to heating the bob or bullet, producing a noise, compacting the bob material, etc.). Instead, we have to conserve momentum. If we denote the velocity of mass m by v and the velocity of mass M by V, using i and f subscripts to mean initially and finally, then:

$$mv_i + MV_i = mv_f + MV_f \tag{1.1}$$

$$m\,(v) + M(0) = m\left(\frac{v}{2}\right) + MV_f \quad \Rightarrow \quad \boxed{V_f = \frac{m}{M}\frac{v}{2}} \tag{1.2}$$

Does this make sense? Well, if the bob is heavier (ie. M is larger) then the speed of the bob goes down. That feels correct, qualitatively. If the bullet is slower (ie. v is smaller), the speed of the bob goes down. Also sensible.

(ii) [3] What is the tension in the rod the moment after the bullet has emerged from the pendulum bob?

The bob will undergo non-uniform circular motion. However, we can apply the circular motion requirement immediately after the bullet leaves because the change in tangential velocity immediately after the strike is almost unnoticeable (it will only change as the bob rises). So, the net force on the bob must be MV_f^2/L. The two forces on the bob are the tension upward and gravity downward. Requiring the sum of the forces to be equal to MV_f^2/L, we find:

$$\frac{MV^2}{L} = T - Mg \tag{1.3}$$

$$T = M\left(\frac{V_f^2}{L} + g\right) \quad \Rightarrow \quad \boxed{T = M\left(\frac{m^2}{M^2}\frac{v^2}{4L} + g\right)} \tag{1.4}$$

Notice the relative signs in the first equation. The centripetal force points towards the pivot (upward), as does the tension, but gravity points downward and thus gets a minus sign.

(iii) [5] What must be the minimum speed of the bullet so that the pendulum bob can swing all the way and complete the circle?

Now we can conserve the energy of the bob, because there will be no heating up, compacting, splintering, etc. Let us denote right after the bullet strike as f and the moment the bob reaches the top of the circle as t. Then, energy conservation reads:

$$E_f = E_t \tag{1.5}$$

$$\text{KE}_f + \text{PE}_f = \text{KE}_t + \text{PE}_t \tag{1.6}$$

where KE denotes kinetic energy and PE denotes potential energy. We can set the bottom of the circle to have no potential energy, because differences in potential energy are all the matter. The potential energy at a height $2L$ (a whole diameter above the bottom of the circle) is $Mg(2L)$. The kinetic energy is always given by $\frac{1}{2}MV^2$:

$$\frac{1}{2}MV_f^2 + 0 = \frac{1}{2}MV_t^2 + Mg(2L) \tag{1.7}$$

If the bob is to *juuuuust* get to the top, then when it gets to the top, $V_t = 0$. To get a complete circle, we must have just slightly more velocity. So, we have

$$\frac{1}{2}MV_f^2 > 2MgL \qquad \Rightarrow \qquad \boxed{V_f^2 > 4gL} \tag{1.8}$$

Plugging in our result from part (i), we find:

$$\left(\frac{m}{M}\frac{v}{2}\right)^2 > 4gL \tag{1.9}$$

$$v^2 > 16gL\left(\frac{M}{m}\right)^2 \qquad \Rightarrow \qquad \boxed{v > 4\frac{M}{m}\sqrt{gL}} \tag{1.10}$$

So, in order to send the bob to the top with a nonzero velocity so that it will complete the circle, we must shoot the bullet with a velocity at least $4\frac{m}{M}\sqrt{gL}$. Does this make sense? If we increase M, then we need to shoot the bullet faster. If we decrease m, then we need to shoot the bullet faster. If the rod is longer, then we need to raise the bob higher, which requires more energy, and thus requires a faster bullet. $\square$