SOLUTION FOR FIRST MIDTERM
PHYSICS 161
Professor R. N. Mohapatra

Administered Friday, February 27^{th}

50 Minutes Allotted.

Solutions by Evan Berkowitz.

50 POSSIBLE POINTS

Two towns A and B are located 80.0 km apart. A couple arranges to drive for a picnic from town A and meet a couple from town B at a lake L. Two couples start to drive from their respective towns simultaneously and drive for 2.50 hours in the directions shown in the figure. The couple from town A drives at a speed of 90.0 km/h. Both the couples arrive simultaneously at the lake L.

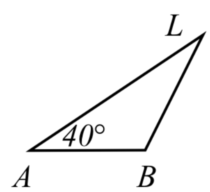

Figure 1: The geography of this problem.

(i) [4] Taking the line joining A to B as the $x-$axis, write the velocity vector of the car 1 (driven by couple from town A) in terms of its components.

Throughout this problem, treat A as the origin. We are told the angle of L with respect to the $x-$axis. If car 1 drives directly from A to L, it must go along that line.

Since we know the *speed* that car 1 drives, we know the *magnitude* of the velocity. Thus, we can write

$$\vec{v} = |v| \left(\cos\theta\widehat{i} + \sin\theta\widehat{j}\right) = \boxed{90\ km/h\ \left(\cos 40^\circ\widehat{i} + \sin 40^\circ\widehat{j}\right)} \tag{1.1}$$

Alternatively, the numerical values:

$$\boxed{68.9\ km/h\ \widehat{i} + 57.85\ km/h\ \widehat{j}} \tag{1.2}$$

(ii) [3] Write the position vector of the lake L in terms of the unit vectors along the $x-$ and $y-$ coordinates as in part (i).

The position of the lake is the same as the position of the car after 2.50 hours. Since the position of the car travels at a constant velocity, we know

$$\vec{x} = \vec{v}t = 90\ km/h\ \left(\cos 40^\circ\widehat{i} + \sin 40^\circ\widehat{j}\right) \cdot 2.5\ h = \boxed{225\ km\ \left(\cos 40^\circ\widehat{i} + \sin 40^\circ\widehat{j}\right)} \tag{1.3}$$

Alternatively, the numerical values:

$$\boxed{172.3\ km\ \widehat{i} + 144.6\ km\ \widehat{j}} \tag{1.4}$$

were accepted.

(iii) [10] What is the speed with which the couple from town B drove?

The location of $B = 80\ km\ \hat{i}$. So, the displacement that car 2 must cover is

$$\Delta\vec{x}_2 = \text{position of } L - \text{position of } B = L_x\hat{i} + L_y\hat{j} - \left(B_x\hat{i} + B_y\hat{j}\right) = (L_x - B_x)\,\hat{i} + (L_y - B_y)\,\hat{j} \quad (1.5)$$

The distance d that car 2 must travel is the magnitude of $\Delta\vec{x}_2$.

$$\begin{aligned} d = |\Delta\vec{x}_2| = \sqrt{(\Delta\vec{x}_2)\cdot(\Delta\vec{x}_2)} &= \sqrt{(L_x - B_x)^2 + (L_y - B_y)^2} && (1.6) \\ &= \sqrt{(172.3\ km\ - 80\ km\ )^2 + (144.6\ km\ - 0\ km\ )^2} \\ &= 171.5\ km && (1.7) \end{aligned}$$

If car 2 travels that distance over 2.5 hours, then the speed s that car 2 must travel is

$$s = \frac{d}{t} = \frac{171.5\ km}{2.5\ h} = \boxed{68.62\ km/h} \quad (1.8)$$

(iv) [3] What is the $x-$component of the velocity of car 2 driven by the couple from town B?

In the previous part we wrote the displacement of car 2 as

$$\Delta\vec{x}_2 = (L_x - B_x)\,\hat{i} + (L_y - B_y)\,\hat{j} \quad (1.9)$$

Assuming the car moves at a constant speed, the velocity of car 2 is

$$\vec{v}_2 = \frac{\Delta x_2}{\Delta t} = \left(\frac{L_x - B_x}{\Delta t}\right)\hat{i} + \left(\frac{L_y - B_y}{\Delta t}\right)\hat{j} \quad (1.10)$$

The $x-$component of the velocity is then

$$v_{2,x} = \frac{L_x - B_x}{\Delta t} = \frac{172.3\ km\ - 80\ km}{2.5h} = \boxed{36.9\ km/h} \quad (1.11)$$

Common Errors:

Error Type (increasing seriousness)	Meaning	Remedy
Mistake in/omission of units	Typically carelessness.	Double check your answers.
Mistake in geometry.	Pythag. theorem, using sine instead of cosine, etc.	Extra care.
Misunderstanding of vectors.	Inability to add vectors or express location of $\vec{L}$	Practice relating vectors to coordinate systems.

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PHYS 161	**Exam 1 - QUESTION 2**
March 3, 2009	**20 POINTS**

A projectile is launched with an initial speed of 60.0 m/s at an angle of 30.0°above the horizontal. The projectile lands on a hillside 4.00 s later. Neglect air friction.

(i) [2] What are the $x-$ and $y-$ components of the projectile's initial velocity.

Since we are told the angle with respect to the horizontal, we can break down the velocity into $x-$ and $y-$ components using its magnitude (ie. the speed) and angle:

$$\vec{v} = v\left(\cos\theta\hat{i} + \sin\theta\hat{j}\right) = 60.0\ m/s\ \left(\cos 30^\circ\hat{i} + \sin 30^\circ\hat{j}\right) \tag{2.1}$$

The accepted answers were

$$\boxed{60.0 \cdot \frac{\sqrt{3}}{2}\ m/s\ \hat{i} + 60 \cdot \frac{1}{2}\ m/s\ \hat{j}} \quad \text{or the numerical values} \quad \boxed{51.96\ m/s\ \hat{i} + 30\ m/s\ \hat{j}}. \tag{2.2}$$

(ii) [3] How high above the ground (the initial launch point) does the projectile climb?

When the projectile is at its highest point, its vertical velocity must be zero (because the derivative at a turning point is 0). Since gravity causes constant acceleration we can use

$$v_{yf}^2 = v_{yi}^2 + 2a_y y \tag{2.3}$$

If we set $v_f = 0$, then y will be the height of the apex. So,

$$y_{top} = -\frac{v_{yi}^2}{2a} = -\frac{(30\ m/s\ )^2}{2 \cdot (-9.8\ m/s^2\ )} = \boxed{45.9\ m} \tag{2.4}$$

If you used $g = -10\ m/s^2$, you would have found

$$\boxed{y_{top} = 45.0\ m} \tag{2.5}$$

(iii) [3] How much time is taken to climb to the highest point after starting?

We know that the projectile will be at the highest point when the vertical component of the velocity is zero. Since gravity causes constant acceleration, we can assert

$$v_y(t) = v_{yi} + at \tag{2.6}$$

where v_{yi} is the initial $y-$component of the velocity and the acceleration $a = -9.8\ m/s^2$ (negative because we have chosen that $\hat{j}$ points upwards). Solving for t we get

$$t = \frac{v_y(t) - v_{yi}}{a} \tag{2.7}$$

To find the time it takes to get to the highest point, we set $v_y(t)$ to 0 and plug in:

$$t_{top} = -\frac{v_{yi}}{a} = -\frac{30\ m/s}{-9.8\ m/s^2} = 3.06\ s \tag{2.8}$$

An alternate method for solving this problem would be to say that

$$y(t) = y_i + v_{yi}t + \frac{1}{2}at^2 \tag{2.9}$$

Where y_i is the initial height (and is thus 0), and $a = -9.8\ m/s^2$ (or $-10\ m/s^2$). Then, to find how long it takes to reach the apex, we could set $y(t)$ to y_{top} from part (ii). We then wind up with

$$y_{top} = 0 + v_{yi}t + \frac{1}{2}at^2 \tag{2.10}$$

Solve this for t to find

$$t = -\frac{v_{yi}}{a} \pm \sqrt{\frac{v_{yi}^2}{a^2} + \frac{2y_{top}}{a}} \tag{2.11}$$

If you compare what is under the square root with the first equality in (2.4), you see that the radical vanishes, so that equation reduces to the first equality in (2.8), reproducing the answer $3.06s$.

A quadratic equation usually has two solutions. Then, why do we only find one value for t? The explanation is as follows: we have set $y(t) = y_{top}$. There is only one time at which $y(t) = y_{top}$, because if there were two times at which that condition were satisfied, then y_{top} would not be the height of the turning point of the parabola. If we set $y(t)$ to a height that was below the apex, then the square-root would not have vanished, and we would find two values for t.

(iv) [3] Using the answer to part (iii) and total time given in the beginning, can you tell whether the landing point of the projectile is in the same horizontal point as the starting point or not? Explain your answer.

We have found that it takes $3.06s$ to reach the top of the arc. So, it should *also* take $3.06s$ to get back to the height of the launch. Therefore, a trip that starts and lands at the same height with the given initial velocity would take $6.12s$. But, we are told in the problem setup that the total flight time is $4.00s$. Therefore, the projectile hasn't had time to come back to the launch height. The point that it hits on the hillside must be at a higher altitude than the launch pad.

(v) [3] What is the projectile's velocity at the highest point in its trajectory?

At the highest point, the velocity of the projectile is purely horizontal. Since gravity only acts vertically, the horizontal velocity must be the same throughout the trip. Therefore,

$$\vec{v}_{top} = \vec{v}_{i,x} = \boxed{51.96\ m/s\ \hat{i}} \tag{2.12}$$

as found in part (i).

(vi) [6] What is the straightline distance from where the projectile was launched to where it hit the ground?

We need to find the displacement of the projectile and compute its magnitude. The horizontal position undergoes no acceleration, and therefore its position is given easily as

$$x(t) = x_i + v_{i,x}t \tag{2.13}$$

The vertical position *does* undergo acceleration, so its position is

$$y(t) = y_i + v_{i,y}t + \frac{1}{2}at^2 \tag{2.14}$$

It is a smart choice to put the launch pad at the origin, so that $x_i = y_i = 0$. The total displacement is given by

$$\vec{x}(t) = x(t)\widehat{i} + y(t)\widehat{j} = (v_{i,x}t)\,\widehat{i} + \left(v_{i,y}t + \frac{1}{2}at^2\right)\widehat{j} \tag{2.15}$$

The magnitude of the displacement is therefore

$$d = \sqrt{\vec{x}(t)\cdot\vec{x}(t)} = \sqrt{(v_{i,x}t)^2 + \left(v_{i,y}t + \frac{1}{2}at^2\right)^2}. \tag{2.16}$$

Plugging in $v_{i,x} = 51.96\ m/s$, $v_{i,y} = 30\ m/s$, $a = -9.8\ m/s^2$ and $t = 4.00s$, we find

$$\boxed{d = 211.9m} \tag{2.17}$$

If you use $a = -10$, you find $\boxed{211.6m}$

Common Errors:

Error (increasing seriousness)	Meaning	Remedy
Mistake in/omission of units	Typically carelessness.	Double check your answers
Incorrect use of formula.	Using the wrong value for a symbol. eg. using the magnitude of the velocity where just the size of the$x$-component is appropriate.	If you memorize a formula (see last common error) try to remember it *in words* instead of in symbols.
Inconsistent coordinate system	If $v_{y0} > 0$ then $g < 0$.	Consider vector nature of quantities and their directions when plugging in #s.
Mistake in geometry.	The straight-line distance includes the vertical displacement.	Careful reading of the question
Mistaken assertion	The peak of the flight *is* a turning point—but *only* in the y-direction. It would be very weird to have an x-turning point	Consider whether the generalization you have made is applicable to all aspects of a situation.
Blindly following a formula.	For instance, the formula $R = \frac{v_0^2 \sin 2\theta}{g}$ has no bearing on this question because it only applies when the projectile lands at launch height.	While studying, do not focus on memorization. Developing an understanding of the problem solving process instead.

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PHYS 161	**Exam 1 - QUESTION 3**
March 3, 2009	**10 POINTS**

(i) [2+2] A rock is thrown horizontally at the same moment that a ball is dropped from the same initial elevation. Which of the two (rock or ball) hits the ground first? Which will have a greater speed?

Since the rock is thrown horizontally, its initial vertical velocity must be 0, just like the ball. Since gravity acts downward independently from the horizontal motion, the two will always have the same vertical velocity, and therefore the two will fall the same distance in the same time. Since they start at the same elevation, they will therefore hit the ground simultaneously.

Even though the vertical velocity of the rock and the ball are the same, the rock has a horizontal velocity whereas the ball does not. The speed is the magnitude of the velocity, and thus the horizontal portion of the rock's velocity will give it a greater speed.

(ii) [3] In uniform circular motion, which of the following quantities are constant: speed, velocity, radial acceleration, tangential acceleration? Which of the quantities are zero throughout the motion?

In uniform circular motion, the speed is constant, which is why it is called *uniform* at all! However, velocity has a directional component, and thus must always be changing. The velocity changes the same way each instant, and thus the radial acceleration must be constant. The tangential acceleration is 0, because tangential acceleration would cause the object to go around faster and faster, which would disrupt the constant-speed requirement.

(iii) [3] A child is sitting inside a car moving at a constant velocity. She throws a ball vertically up. Can she catch the ball without moving her body as it falls? If the car is moving at a constant acceleration, does the answer change? If yes, why and if no, why not?

If she throws the ball up vertically in the constant-velocity car, the ball will land in her hand without her moving. However, if the car is accelerating then once the ball leaves the girl's hand, there will be no force on the ball, and so it will go at a constant horizontal velocity, while the girl's velocity changes. Thus, the two at any time will have a relative velocity, meaning that the girl will have to reach out to catch the ball (if the car is decelerating, she will reach forward and if the car is accelerating, she will have to move her hand backward).

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