SOLUTION FOR SECOND MIDTERM
PHYSICS 161
Professor R. N. Mohapatra
Administered Friday, April 3^{rd}
50 Minutes Allotted.
Solutions by Evan Berkowitz.

50 POSSIBLE POINTS

Consider the three connected objects in the figure provided. The masses on the inclined plane are 1.0 and 2.0 kg respectively and that of the hanging block is M. The inclined plane is frictionless, the pulley and strings are massless and the system is in equilibrium.

For our purposes, label the blocks from left to right 1, 2, 3. So, the block with mass 2.0 kg is block 1, the block with mass 1.0 kg is block 2, and the hanging mass is mass 3. For our purposes, let us call the angle that the incline makes with the horizontal θ, which we happen to know is 30°. Let us simplify our problems by using $g = 10\ m/s^2$ instead of 9.8 m/s^2 .

(a) [6] Draw the free body diagram for the three blocks.

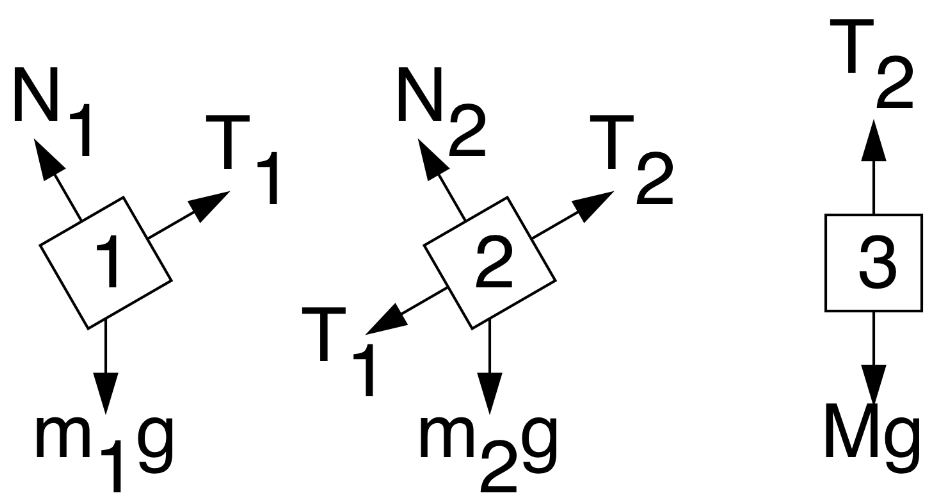

Figure 1: Free body diagrams

The forces labelled with a T are tension forces, the forces labelled with N are normal forces, provided by the incline. The tensions are parallel to the surface, which is at an angle $\theta = 30°$ to the horizontal.

(b) [6] Applying Newton's laws to each block, find the mass of the hanging block, M.

A quick way to do this problem is to consider block 1 and block 2 as one compound object, with mass $m_1 + m_2$. This compound object is held in place by T_2, which is fighting against the parallel component of the gravity. Breaking the gravitational force into components, as in 2, tells us that the total gravitational force on the compound object is just its mass times gravity times the sine of the angle of the incline to the left. Equilibrium requires that T_2 balance out that force. Since it points to the up-right where connected to the blocks, we must have:

$$T_2 = (m_1 + m_2) g \sin(\theta) \tag{1.1}$$

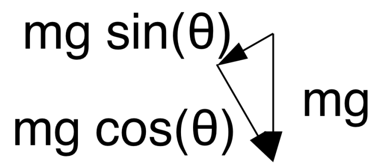

Figure 2: Parallel and perpendicular components of the gravitational force.

Another requirement of equilibrium is that the hanging block not move. The net force on it must be zero. Then, from block 3's diagram we immediately see:

$$T_2 = Mg \tag{1.2}$$

Equating our two expressions for T_2, we find:

$$Mg = (m_1 + m_2) g \sin(\theta) \qquad \Rightarrow \qquad \boxed{M = (m_1 + m_2)\sin(\theta)} \tag{1.3}$$

Plugging our numbers in, we find

$$M = (1\ kg\ + 2\ kg\ )\sin(30°) = 3\ kg\ \times \frac{1}{2} = \boxed{1.5 kg} \tag{1.4}$$

(c) [4] Find the tensions in each of the strings.

To find T_2 we can use (1.2). Plugging in numbers we find

$$T_2 = 1.5\ kg\ 10\ m/s^2\ = \boxed{15N} \tag{1.5}$$

To find T_1 we can use the block diagram of either block 1 or block 2.

Using block 1, we must balance T_1 with the parallel component of the force of gravity on block 1. Using Fig. 2 to get the parallel component of the gravity, we then can say

$$T_1 = m_1 g \sin\theta \tag{1.6}$$

Plugging numbers into this we find

$$T_1 = 2\ kg\ 10\ m/s^2\ \times \frac{1}{2} = \boxed{10N} \tag{1.7}$$

Using block 2, we must balance T_1 and the parallel component of the force of gravity on block 2 against the force supplied by T_2. Thus,

$$T_1 + m_2 g \sin\theta = T_2 \qquad \Rightarrow \qquad T_1 = T_2 - m_2 g \sin\theta \tag{1.8}$$

Plugging numbers into this we find

$$T_1 = 15N - 1\ kg\ 10\ m/s^2\ \times \frac{1}{2} = 15N - 5N = \boxed{10N} \tag{1.9}$$

which matches what we found by considering the forces on block 1. If we did *not* find the same number using the forces on block 2 as we found using the forces on block 1, we'd have a problem, because the tension in a massless string must be the same throughout the string.

(d) [3+3] If the inclined plane, instead of being frictionless, had coefficients of static and kinematic friction 0.50 and 0.30 respectively, then what would the maximum and minimum values of the mass M for which the system would be at rest. Draw a new set of free body diagrams as you solve this problem.

Again, let's just lump blocks 1 and 2 into a compound object. The compound object has a free body diagram as follows: Notice how the action-reaction pair does not effect the motion of the

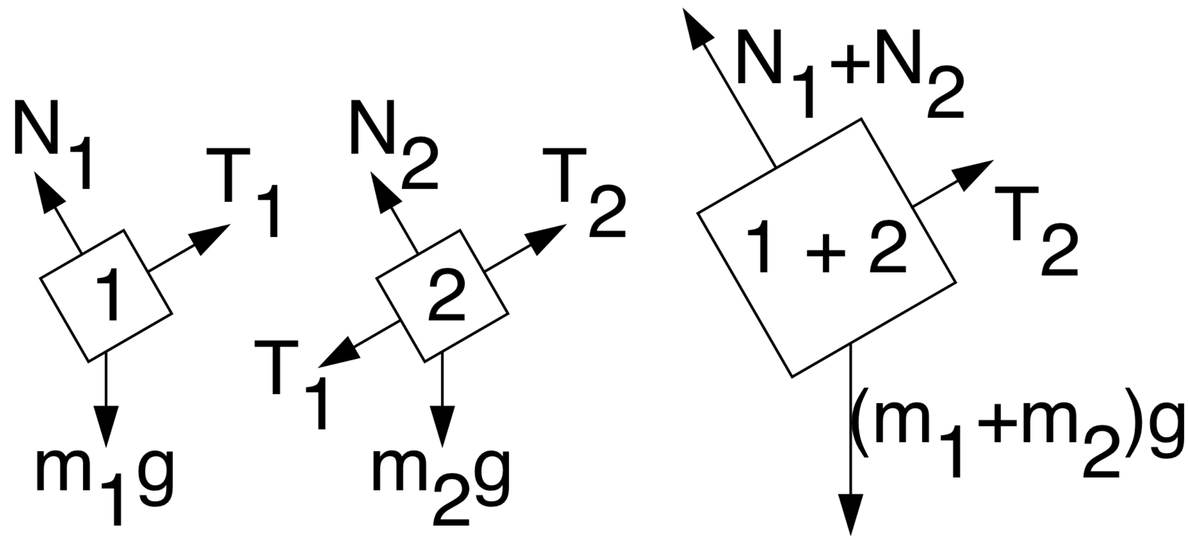

Figure 3: Combining the free-body diagrams

compound object. We have left out the friction from this free-body diagram, because we don't know which way it points. A proper diagram might have it included. Let's consider the other parallel forces on the object. If up-and-right is positive, then the forces we have drawn have a total parallel component equal to

$$T_2 - (m_1 + m_2)g\sin\theta \tag{1.10}$$

If T_2 is more than $(m_1 + m_2)g\sin\theta$ then the friction will point in the negative parallel direction (ie. against the motion). If $(m_1 + m_2)g\sin\theta$ is more than T_2, then the friction will point in the positive parallel direction (ie. still against the motion). The friction can be as large as $\mu_s N$. So, since we require that the size of the other forces be less than or equal to the size of the friction to be in static equilibrium,

$$|T_2 - (m_1 + m_2)g\sin\theta| \leq |f| \tag{1.11}$$

This breaks into two cases:

$$(m_1 + m_2)g\sin\theta - T_2 \leq f \quad \text{and} \quad T_2 - (m_1 + m_2)g\sin\theta \leq f \tag{1.12}$$

which give

$$(m_1 + m_2)g\sin\theta - f \leq T_2 \quad \text{and} \quad T_2 \leq (m_1 + m_2)g\sin\theta + f \tag{1.13}$$

We know, for equilibrium, T_2 must be Mg, by block 3's free body diagram. Also, we know that the friction will be $f = \mu_s N = \mu_s(m_1 + m_2)g\cos\theta$. Plugging those expressions in, we find

$$(m_1 + m_2)g\sin\theta - \mu_s(m_1 + m_2)g\cos\theta \leq \quad Mg \quad \leq \quad (m_1 + m_2)g\sin\theta + \mu_s(m_1 + m_2)g\cos\theta \tag{1.14}$$

Cancelling out the gs and factoring the $(m_1 + m_2)$ out, we find

$$(m_1 + m_2)(\sin\theta - \mu_s \cos\theta) \leq \quad M \quad \leq \quad (m_1 + m_2)(\sin\theta + \mu_s \cos\theta) \tag{1.15}$$

Plugging in the numbers, we find

$$(2\ kg\ + 1\ kg\ )(\sin 30^\circ - 0.5 \cos 30^\circ) \leq \quad M \quad \leq \quad (2\ kg\ + 1\ kg\ )(\sin 30^\circ + 0.5 \cos 30^\circ) \tag{1.16}$$

$$(3\ kg\ )\left(\frac{1}{2} - 0.5\frac{\sqrt{3}}{2}\right) \leq \quad M \quad \leq \quad (3\ kg\ )\left(\frac{1}{2} + 0.5\frac{\sqrt{3}}{2}\right) \tag{1.17}$$

Finally,

$$\boxed{0.20\ kg\ \leq M \leq 2.8\ kg} \tag{1.18}$$

□

PHYS 161	**Exam 2 - QUESTION 2**
April 7, 2009	**16 POINTS**

A roller-coaster has a mass of 500 kg when fully loaded with passengers. It rides on a track, provided. Point A is at the bottom of a dip, and has a radius of curvature $R_A = 10.0\ m$. Point B is at the top of a hill, and has a radius of curvature $R_B = 15.0\ m$

Let us denote the mass of the roller-coaster as m. We are told $m = 500\ kg$. We will use $g = 10\ m/s^2$ instead of $9.8\ m/s^2$.

(a) [4] Draw the free body diagrams for the roller-coaster when it is at the points A and B.

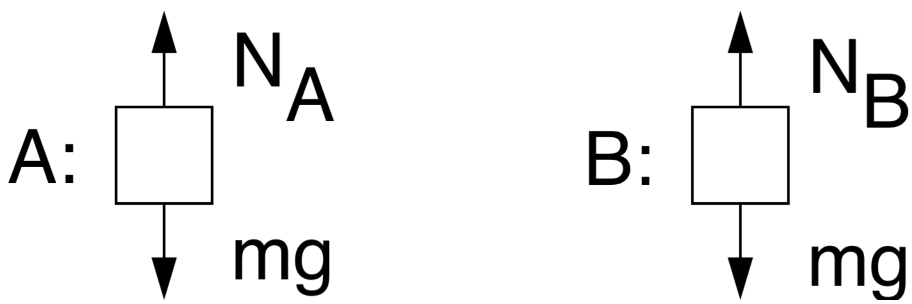

Figure 4: The free body diagrams at points A and B.

(b) [6] If the car has a speed of $20\ m/s$ when it is at point A, what is the force exerted by the track on the car at that point?

When it is at A, it seems to the car that it is undergoing uniform circular motion in a circle of radius R_A. Thus, we know that the net force must point towards the center of the circle (upwards) and have magnitude mv^2/r. Let us denote this by saying

$$\sum \vec{F} = \frac{mv_A^2}{R_A}\widehat{y} \tag{2.1}$$

choosing the $\widehat{y}$ direction to be upwards, and $v_A = 20\ m/s$ is the velocity at point A.

But, we can compute the sum of the forces. We can see from the free body diagram that it is

$$\sum \vec{F} = N\widehat{y} - mg\widehat{y} \tag{2.2}$$

Equating our expressions for the sum of the forces, we find

$$N\widehat{y} - mg\widehat{y} = \frac{mv_A^2}{R_A}\widehat{y} \qquad \Rightarrow \qquad N\widehat{y} = m\left(\frac{v_A^2}{R_a} + g\right)\widehat{y} \tag{2.3}$$

Solving for N, we find

$$N = m\left(\frac{v_A^2}{R_a} + g\right) \tag{2.4}$$

Plugging in our numbers, we find

$$N = 500\ kg\ \left(\frac{(20\ m/s\ )^2}{10m} + 10\ m/s^2 \right) = 500kg \times 50\ m/s^2\ = \boxed{25000N} \tag{2.5}$$

(c) [6] What is the maximum speed the car can have at the point B and still remain in contact with the track?

At point B, the car seems to be undergoing uniform circular motion in a circle of radius R_B. Thus, we know that the net force must point towards the center of the circle (downwards this time!) and have a magnitude mv^2/r. Let us denote this by saying

$$\sum \vec{F} = -\frac{mv_B^2}{R_B}\widehat{y} \tag{2.6}$$

choosing the $\widehat{y}$ direction to be upwards, so that the right hand side is negative and thus points downwards.

But, we can compute the sum of the forces from our free body diagram. We can see

$$\sum \vec{F} = N\widehat{y} - mg\widehat{y} \tag{2.7}$$

Equating our expressions for the sum of the forces, we find

$$-\frac{mv_B^2}{R_B}\widehat{y} = N\widehat{y} - mg\widehat{y} \qquad \Rightarrow \qquad v_B^2\widehat{y} = \left(g - \frac{N}{m}\right) R_B\widehat{y} \tag{2.8}$$

We can drop the $\widehat{y}$ from both sides, because the magnitudes of the components must be equal.

Now, we are asked what the maximum speed the car can have so that it just remains in contact with the track. Suppose the car just barely leaves the track at point B. Then, since there is no contact, N must be zero. So, the fastest the car can go without leaving the track is the speed that *juuuuust* makes N zero. Therefore, we find:

$$v_B^2 = gR_B \qquad \Rightarrow \qquad v_B = \sqrt{gR_b} \tag{2.9}$$

Plugging in our numbers, we find

$$v_B = \sqrt{10\ m/s^2\ 15.0m} = \sqrt{150\,(\ m/s\ )^2} = \boxed{5\sqrt{6}\ m/s\ \approx\ 12.24\ m/s} \tag{2.10}$$

□

PHYS 161	**Exam 2 - QUESTION 3**
April 7, 2009	**12 POINTS**

(a) [2] A car runs out of gas while driving down a hill. It rolls through a valley and starts up the other side. At the very bottom of the hill, which of the free body diagrams is the correct one in the provided figure?

The "circle" that the car is passing through is above it, so requiring that the net force point towards its center means that all of the forces should be vertical (ruling out choices 4, 5, and 6) and that the sum of the forces should point upwards (ruling out 2 and 3), so the correct choice is 1.

(b) [2] You are standing on one end of a light wooden raft that has floated 3m away from the pier, as shown in the provided figure. You are standing on the raft end nearest to the pier. How can you propel it towards the pier without getting off the raft? Explain your answers using Newton's laws.

If you step to the right (away from the pier), then the frictional force on your shoes must be to the right, because that's the direction you have accelerated in, and $\vec{F} = m\vec{a}$ (Newton's second law). So, the frictional force of the raft on your shoes is to the right (away from the pier). This means that the frictional force of your shoes on the raft is to the left (towards the pier), because it must point oppositely to the force on your shoes, by Newton's third law.

(c) [2] Three boxes are being pushed across a frictionless horizontal surface as shown in the provided figure. Let $N_{2,5}$ denote the magnitude of the normal force exerted by the box of mass 2kg on the box of mass 5kg and $N_{5,10}$ denote the magnitude of the normal force exerted by the box of mass 5kg on the box of mass 10kg.

Which of the relations provided hold if all the boxes are always in contact and moving with an acceleration to the right; give a brief explanation of your answer.

If F (the applied force) is equal in magnitude to $N_{2,5}$, then by Newton's third law, F is also equal in magnitude to $N_{5,2}$. That would mean that the net force on the block of mass 2kg is zero, and thus that that block would not accelerate. But we are told that it *does* accelerate to the right (the direction F pushes), so it must be that $F > N_{5,2}$ and therefore $F > N_{2,5}$.

Now let's use that same argument to compare $N_{2,5}$ to $N_{5,10}$. First, note that $N_{5,10} = N_{10,5}$ but points in the opposite direction. If $N_{10,5}$ were equal to $N_{2,5}$ then the total force on block of mass 5kg would be zero, and thus it would not speed up. But, we are told that it *does* speed up to the right (the direction $N_{2,5}$ points), so it must be that $N_{2,5} > N_{10,5}$ and therefore $N_{2,5} > N_{5,10}$. So, we should chose option (iii) $F > N_{2,5} > N_{5,10}$.

(d) [2] You are in a moving elevator with a bathroom scale. You know how much you weigh. Using the scale, can you tell how fast the elevator is accelerating? Can you tell the speed of the elevator, if it is moving at a uniform speed?

Bathroom scales work by measuring how much normal force they apply. Thus, if they apply a force

N to you, the dial would read N.

If the elevator is moving at a uniform speed, then *you* are moving at a uniform speed, and so the net force on you must be zero. Thus, N must be mg and you cannot distinguish this experiment from an elevator that is still (or from an elevator that is moving at any other uniform speed.)

However, if the elevator is accelerating, *you* are accelerating, so the net force on you must be nonzero. Suppose the elevator is accelerating upwards. Then, the net force on you is upwards, and so N will be more than mg, so the scale would read more than your weight. Suppose the elevator is accelerating downwards. Then, the net force on you is downwards, and so N will be less than mg, and so the scale would read less than your weight. Either way, if the elevator is accelerating, you can tell because the scale does not read your weight.

(e) [2] Assume that the Earth is a sphere. Will the apparent weight of a person be more at the poles or the equator? Why?

If the Earth is a sphere, the force of gravity on its surface is a constant (ie. $\vec{g}$ has a constant magnitude). A person at the poles feels this force downward, and the normal force that the floor must apply to them is equal in magnitude. So, at the poles, the apparent weight of the person is mg. A person at the equator undergoes circular motion. Therefore, the net force on that person must be toward the center of the circle. The only two forces on that person are mg towards the center of the Earth and N the normal force away from the center of the Earth. If the net force is towards the center of the Earth, mg must be larger than N, so $N < mg$. Therefore, the apparent weight of a person on the equator is *less* than the apparent weight of the same person at the poles.

(f) [2] A television is sitting on a table. Write down all the forces acting on the television and the table and identify the action-reaction pairs for all the forces in this system. The arrows are

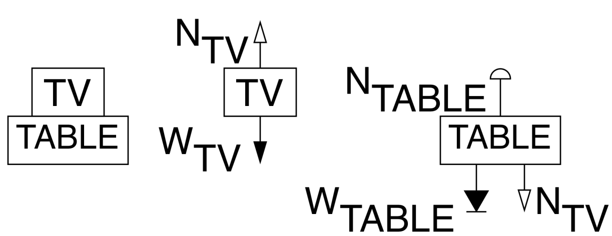

Figure 5: The free body diagrams.

drawn with different heads to denote they are part of different action-reaction pairs. Forces that are denoted N are normal forces. Forces that are denoted W are weights (forces of gravity). The only drawn action-reaction pair is the normal force that the TV and the table share, labelled N_{TV}. W_{TV}, W_{TABLE}, and N_{TABLE} *are* part of action-reaction pairs, but we have not drawn the object that their partners act on: the Earth!

There are some forces that are *NOT* action-reaction pairs that *happen* to be equal because the objects are in equilibrium. That is, $N_{TV} = W_{TV}$, and $N_{TABLE} = W_{TABLE} + N_{TV}$, but these equalities are not from Newton's 3rd law, they are from Newton's 2nd law (with $\vec{a} = 0$). □